Extraction of Transport Coefficients in Complex Production Networks

IPAM – KT WORKSHOP 3
"Flows and Networks in Complex Media"
April 27- May 1, 2009

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Introduction

- Overall Goal: To create a model from observations of a given system without knowledge of the obtained internal mechanisms and to find a computationally efficient way to simulate production systems.
- Traffic flow type models are used for production systems. All products enter the system at $x = 0$ and leave the system at $x = 1$.
- Stage of processes is considered as an artificial "space" (road) in the simulations.

Structure of Toy Factory Model

	diffusion1	diffusion2	litho1	etch clean	etch1	ion impl	metal dep	litho2	etch2	
step	a	b	c	d	e	f	g	h	i	
1				0.25						clean wafer
2	8.00									grow a layer
3			1.00							pattern it
4					1.00					etch away some
5		6.00								grow a layer
6			1.25							pattern it
7						2.50				implant ions
8				0.50						remove mask
9	7.00									grow a layer
10			1.00							pattern it
11					1.00					etch some away
12				0.25						clean wafer
13		5.00								grow a layer
14			1.25							pattern it
15						3.50				implant ions
16				0.50						remove mask
17								1.50		pattern contact
18									1.75	etch contact
19							2.25			layer metal
20								1.00		pattern metal
21									2.25	etch metal
22								1.50		pattern contact
23									2.00	etch contact
24							2.25			layer metal
25								1.00		pattern metal
26									2.50	etch metal
200										lots started per week

Structure of Toy Factory Model

diffusion1	diffusion2	litho1	etch clean	etch1	ion impl	metal dep	litho2	etch2	
15.00	11.00	4.50	1.50	2.00	6.00	4.50	5.00	8.50	total hours required per lot
3000	2200	900	300	400	1200	900	1000	1700	total hours needed per week
0.80	0.75	0.90	0.70	0.75	0.85	0.85	0.90	0.65	(average availability)
134.4	126.0	151.2	117.6	126.0	142.8	142.8	151.2	109.2	total hours available per machine per week
22.32	17.46	5.95	2.55	3.17	8.40	6.30	6.61	15.57	tools needed as time required/time available
1.25	1.25	1.00	2.00	1.50	1.25	1.25	1.10	1.50	desired degree of constrainedness
27.90	21.83	5.95	5.10	4.76	10.50	7.88	7.28	23.35	number of tools needed
28	**22**	**6**	**6**	**5**	**11**	**8**	**8**	**24**	number of tools installed

Macroscopic State Variable

- We also record the value of macroscopic state variables (Z_n^m) at each time a_n^m accordingly and get a Z table similar to time table given as follows:

Lots	S_1	S_2	S_3	…	S_M	(Stages)
1	$Z_{1,1}$	$Z_{1,2}$	$Z_{1,3}$	…	$Z_{1,M}$	at times $a_{1,1}$ … $a_{1,M}$
2	$Z_{2,1}$	$Z_{2,2}$	$Z_{2,3}$	…	$Z_{2,M}$	at times $a_{2,1}$ … $a_{2,M}$
	…	…	…	…	…	…
N	$Z_{N,1}$	$Z_{N,2}$	$Z_{N,3}$	…	$Z_{N,M}$	at times $a_{N,1}$ … $a_{N,M}$

- Z's can be variables like the total WIP, upstream WIP, downstream WIP, or just the stage index m itself.

Objectives

1) To extract the transport coefficients (C and D) from the observed data out of a given system (black box) and numerically solve the PDE:

$$\partial_t \rho + \partial_x F = 0, \;\; F(x) = C\rho - D\partial_x \rho$$

2) With the Traffic flow-like PDE model, we predict the transient (non-steady state) behavior of the given model according to the same influx. Do the behaviors of the two models agree with each other?

PDE - Conservation Law

- We aim to solve the following PDE (Partial Differential Equation) – conservation law:

  $\partial_t \rho + \partial_x [C\rho - D\partial_x \rho] = 0$, where

  $F(x,t) = C\rho - D\partial_x \rho$: Flux function,

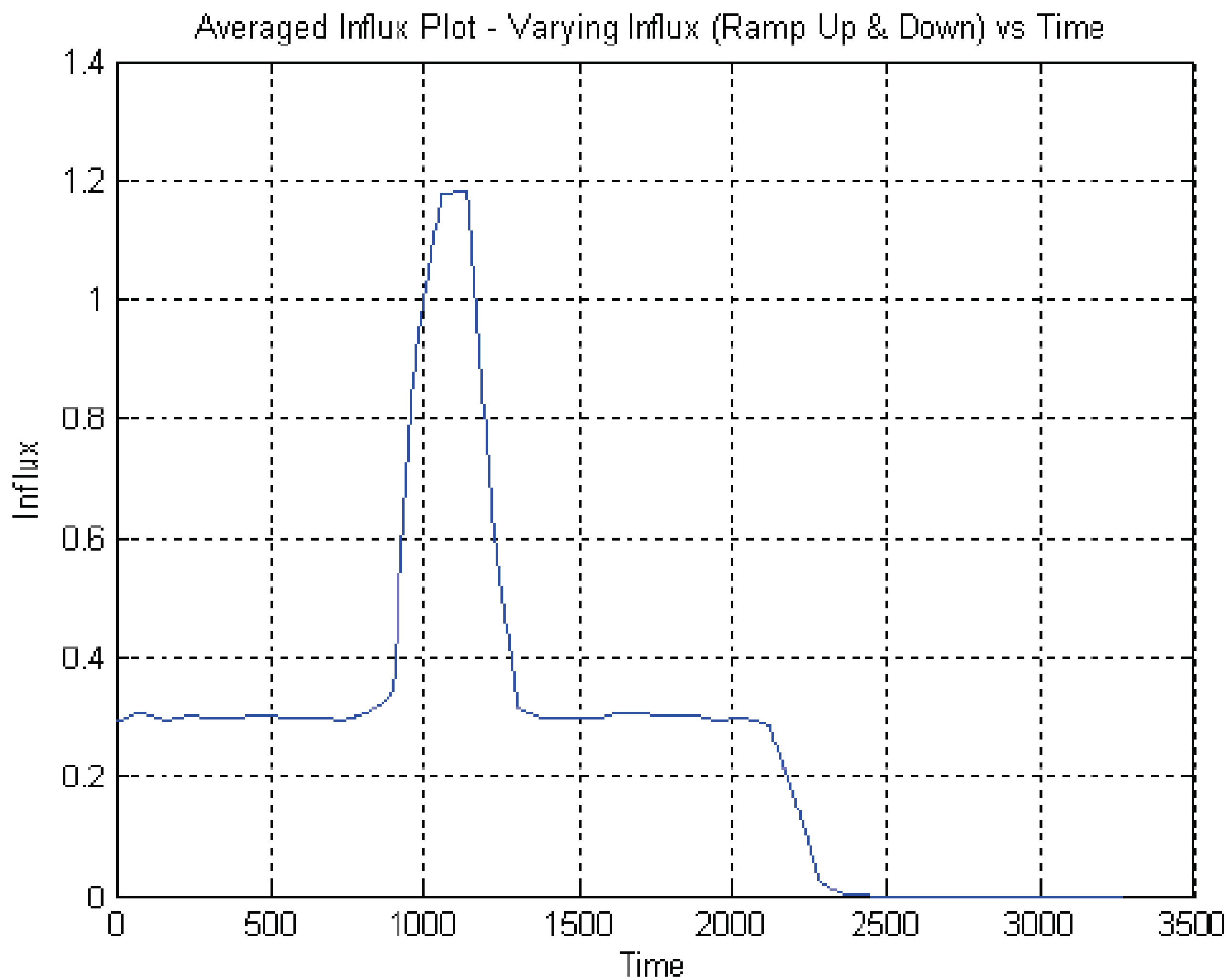
  C: Convection (Velocity) coefficient,

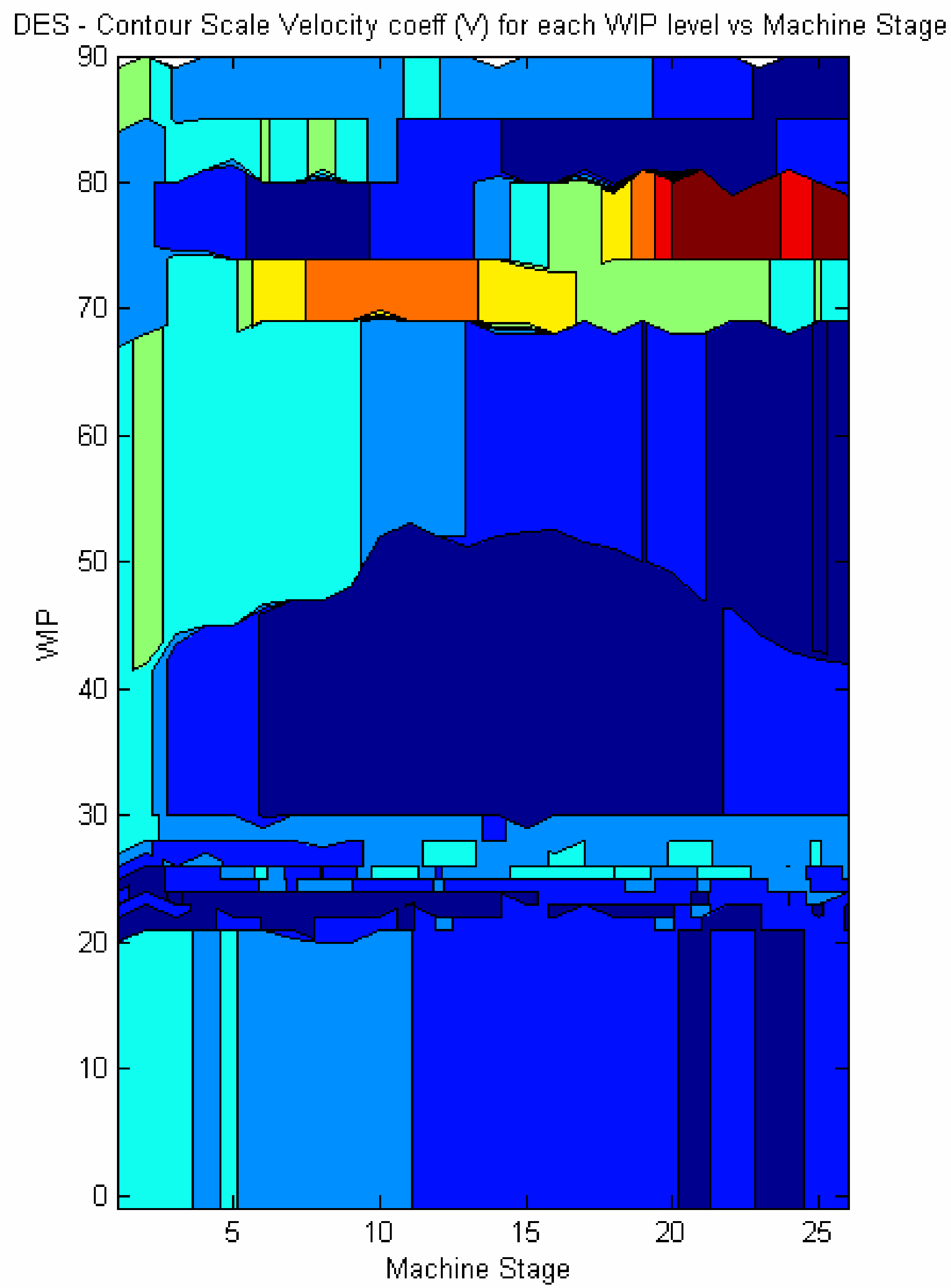
  D: Diffusion coefficient,

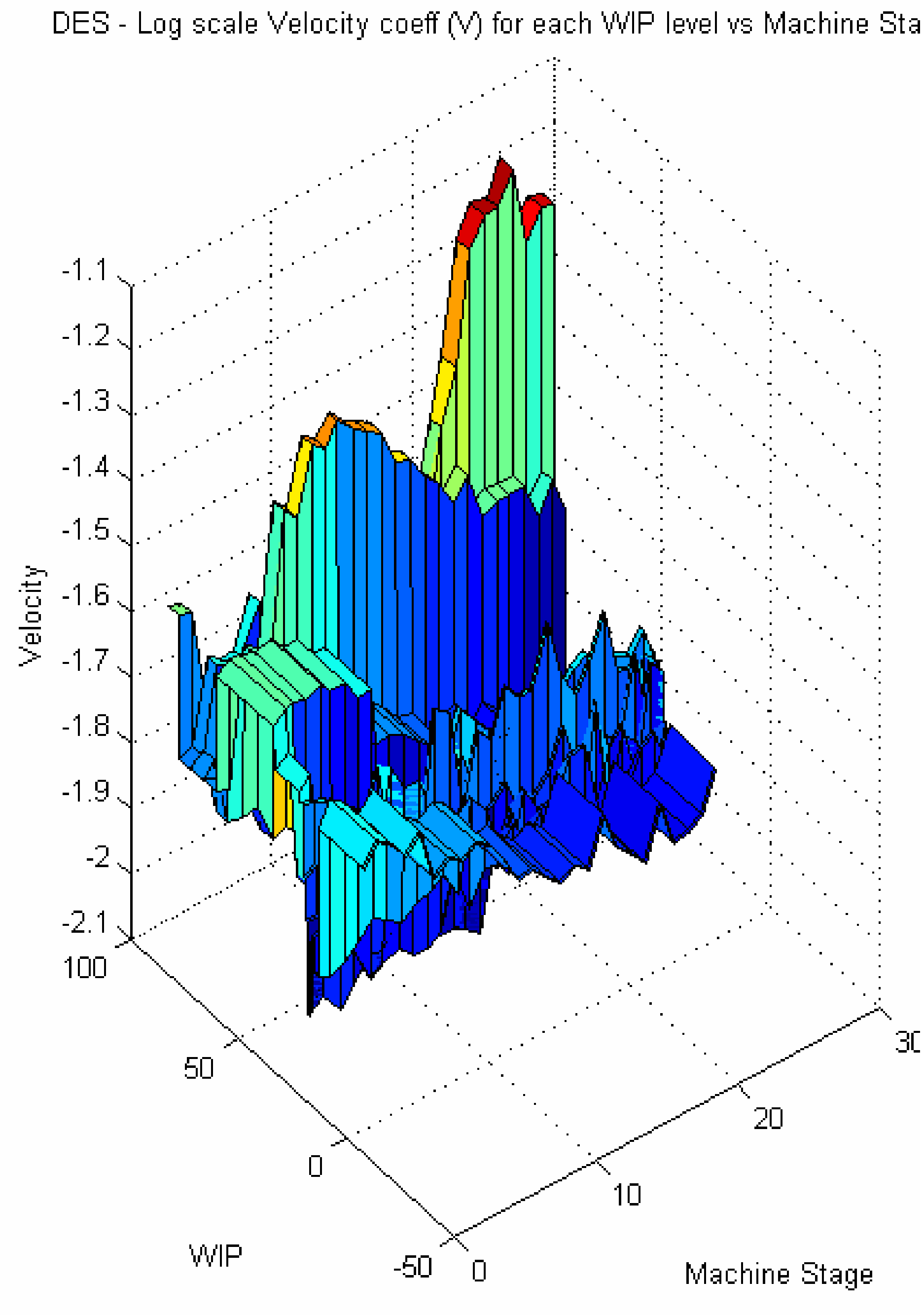
  ρ: Density of particles ($\rho = \rho(x,t)$).

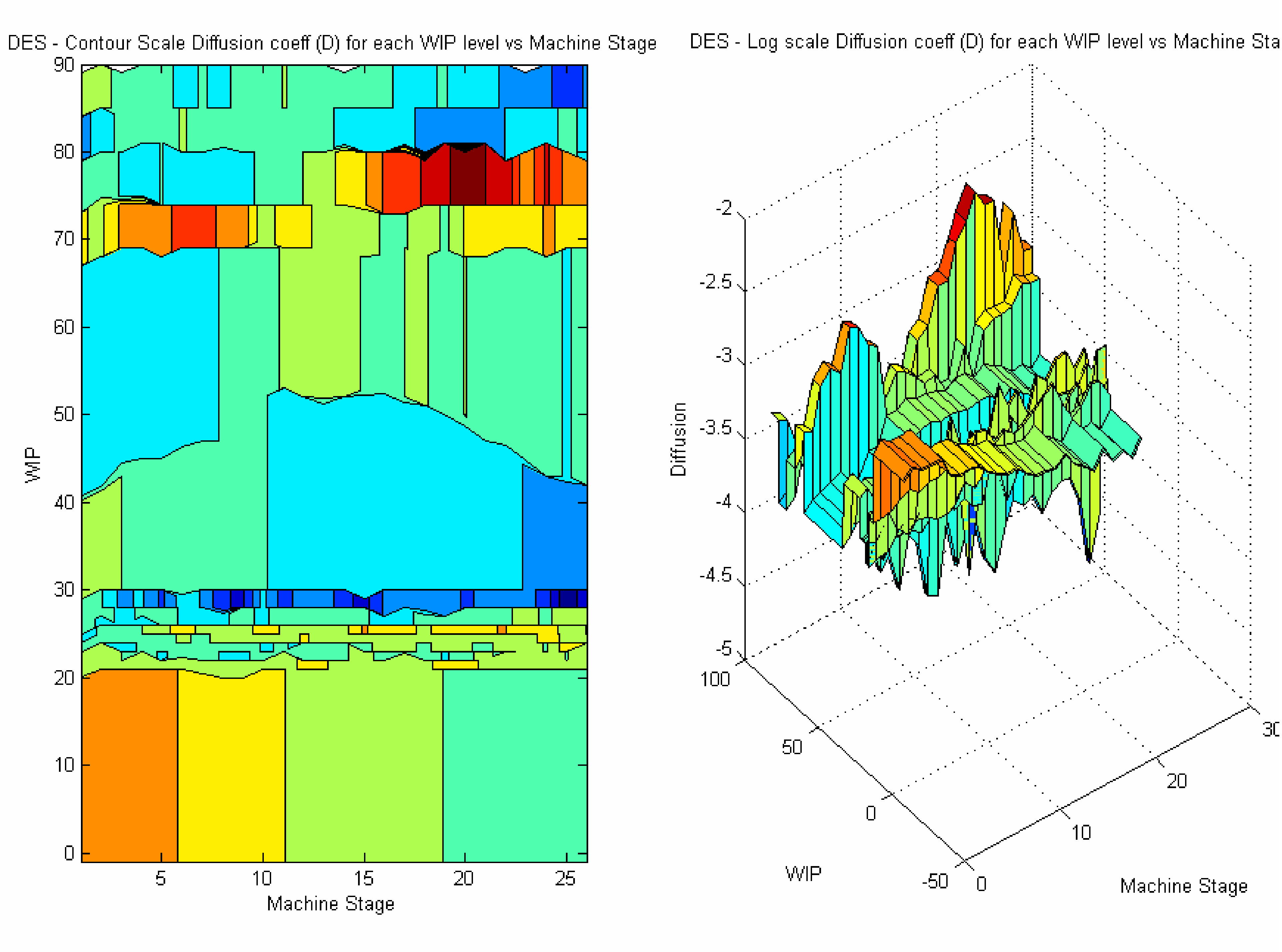
- Here, $\rho(x,t)$ is the local WIP (Work In Progress) density of products $[\text{parts/space}]$ at stage x and at time t.

Averaged Varying Influx

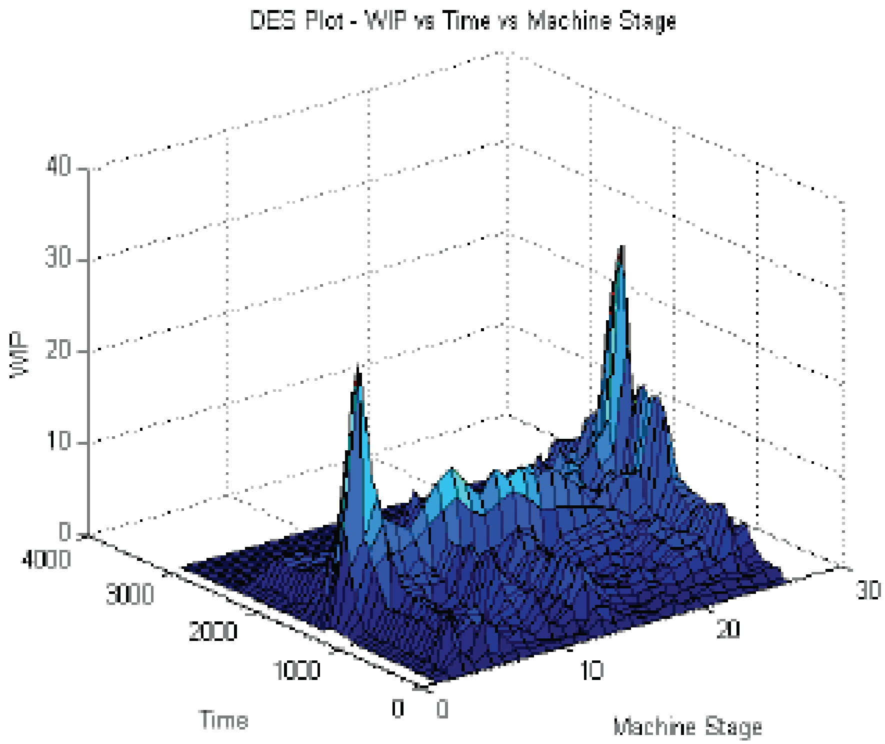


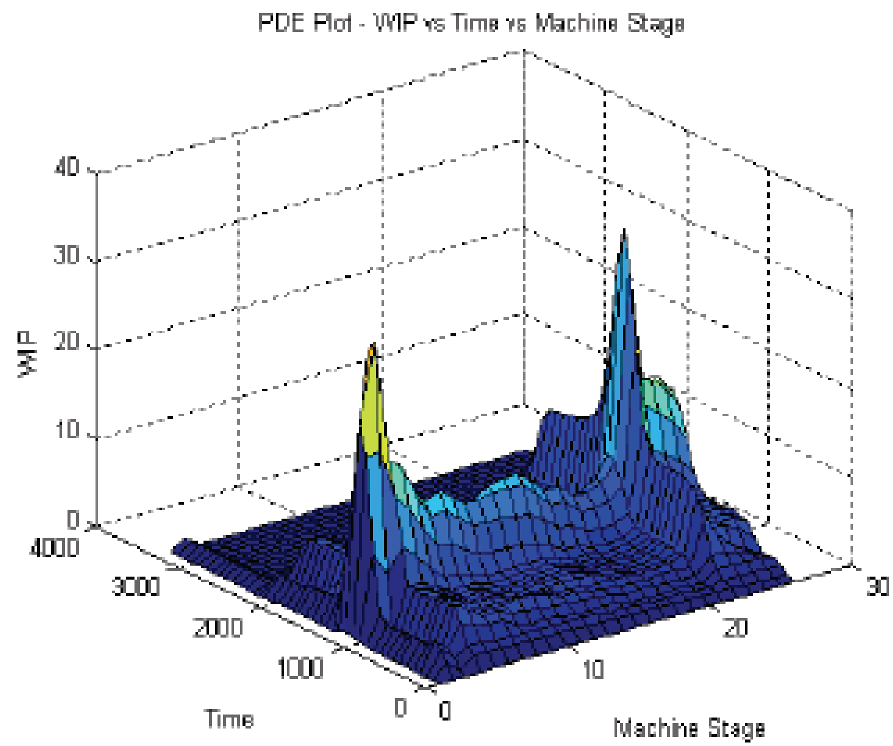

DES - Contour Scale Velocity coeff (V) for each WIP level vs Machine Stage
WIP
Machine Stage
0
10
20
30
40
50
60
70
80
90
5
10
15
20
25


DES - Log scale Velocity coeff (V) for each WIP level vs Machine Sta
Velocity
WIP
Machine Stage
-1.1
-1.2
-1.3
-1.4
-1.5
-1.6
-1.7
-1.8
-1.9
-2
-2.1
100
50
0
-50
0
10
20
30

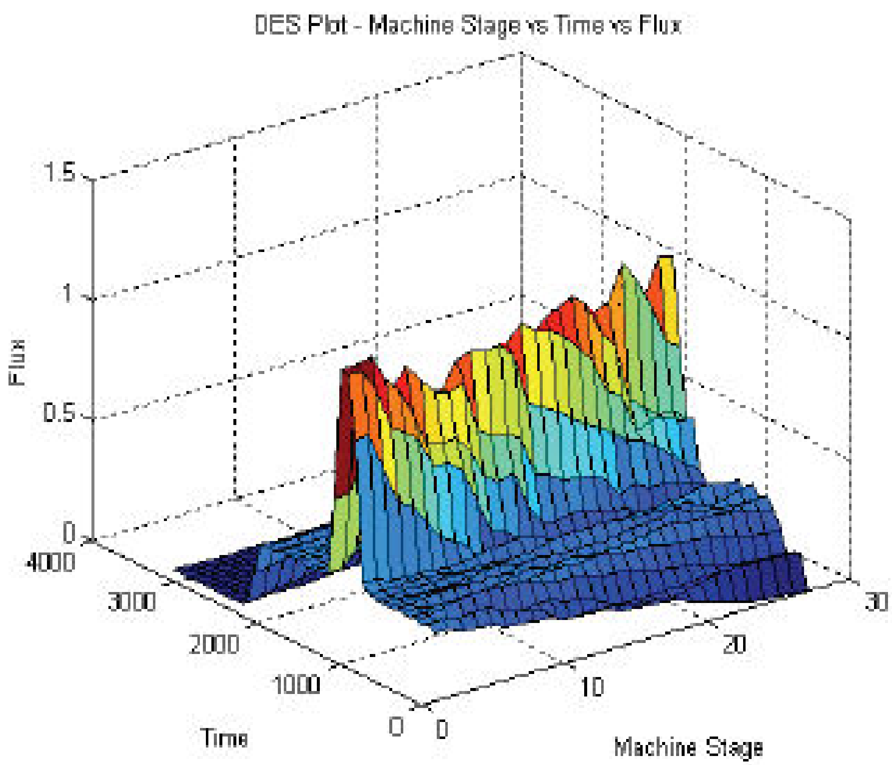

DES - Contour Scale Diffusion coeff (D) for each WIP level vs Machine Stage
WIP
90
80
70
60
50
40
30
20
10
0
5
10
15
20
25
Machine Stage
DES - Log scale Diffusion coeff (D) for each WIP level vs Machine Sta
Diffusion
-2
-2.5
-3
-3.5
-4
-4.5
-5
100
50
0
-50
WIP
0
10
20
30
Machine Stage

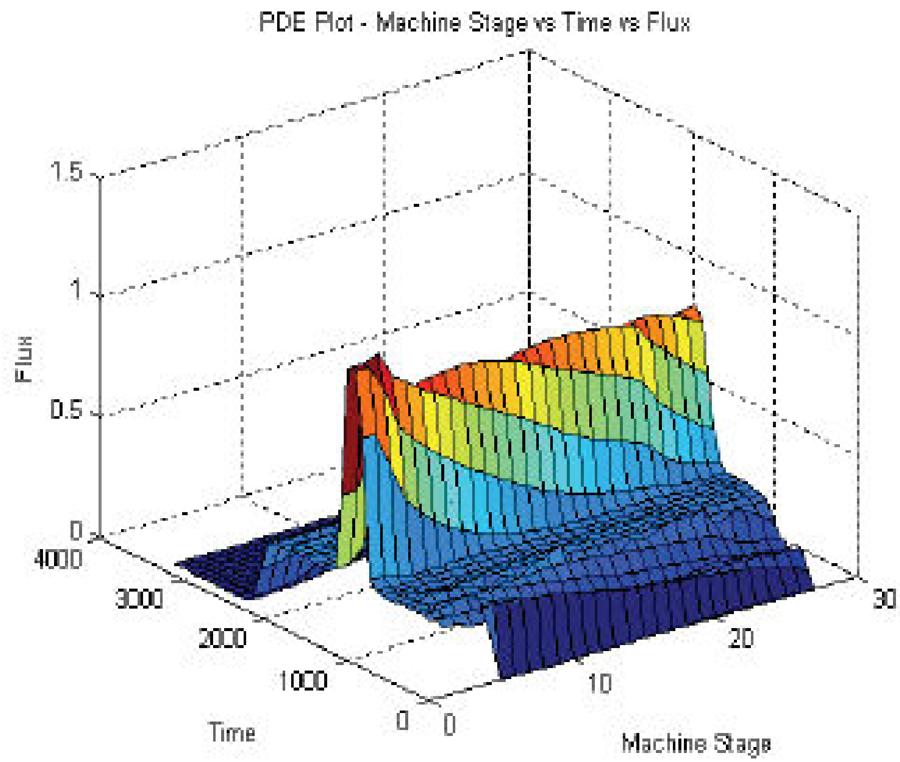
Comparison of WIP Levels in 3D

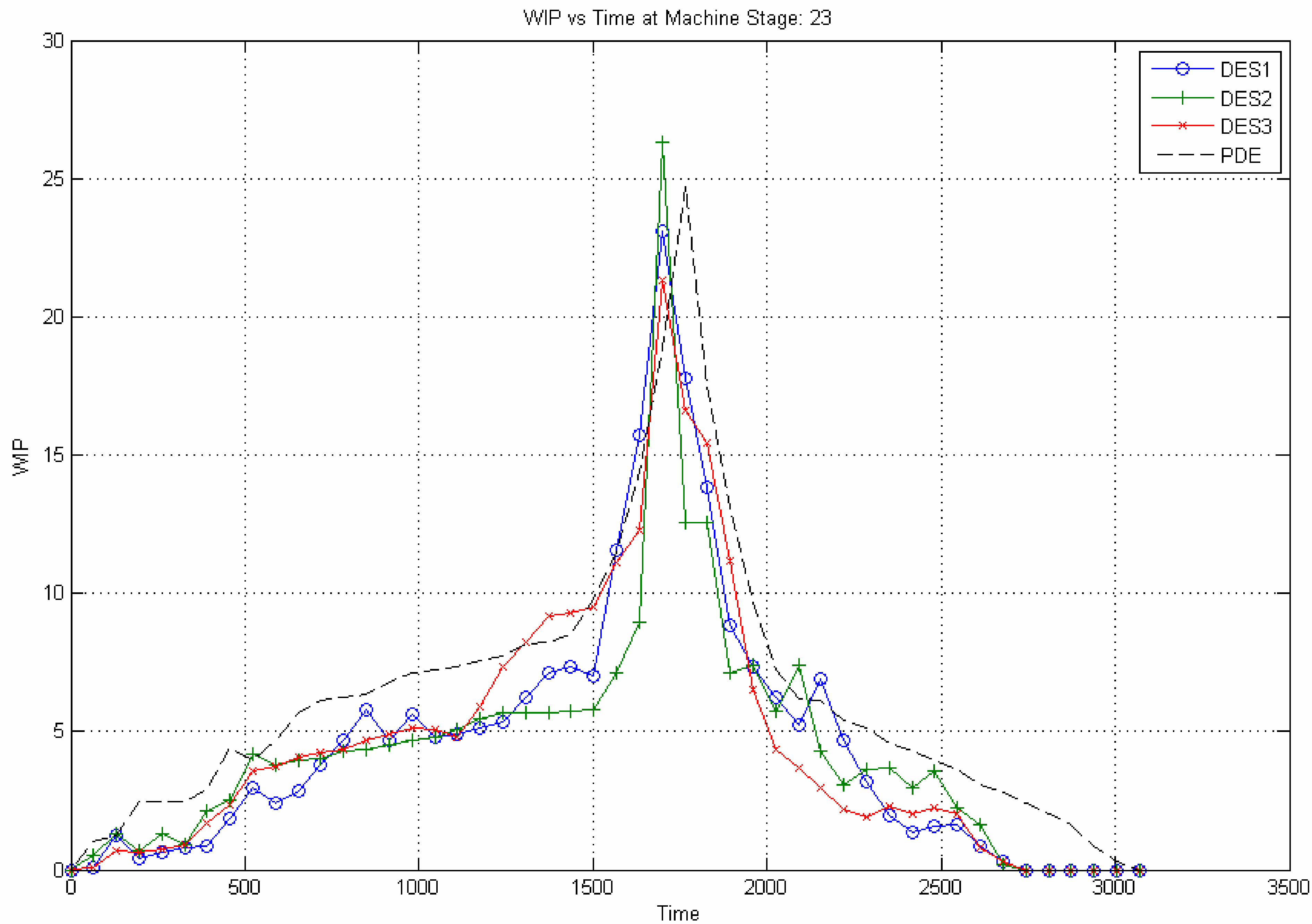


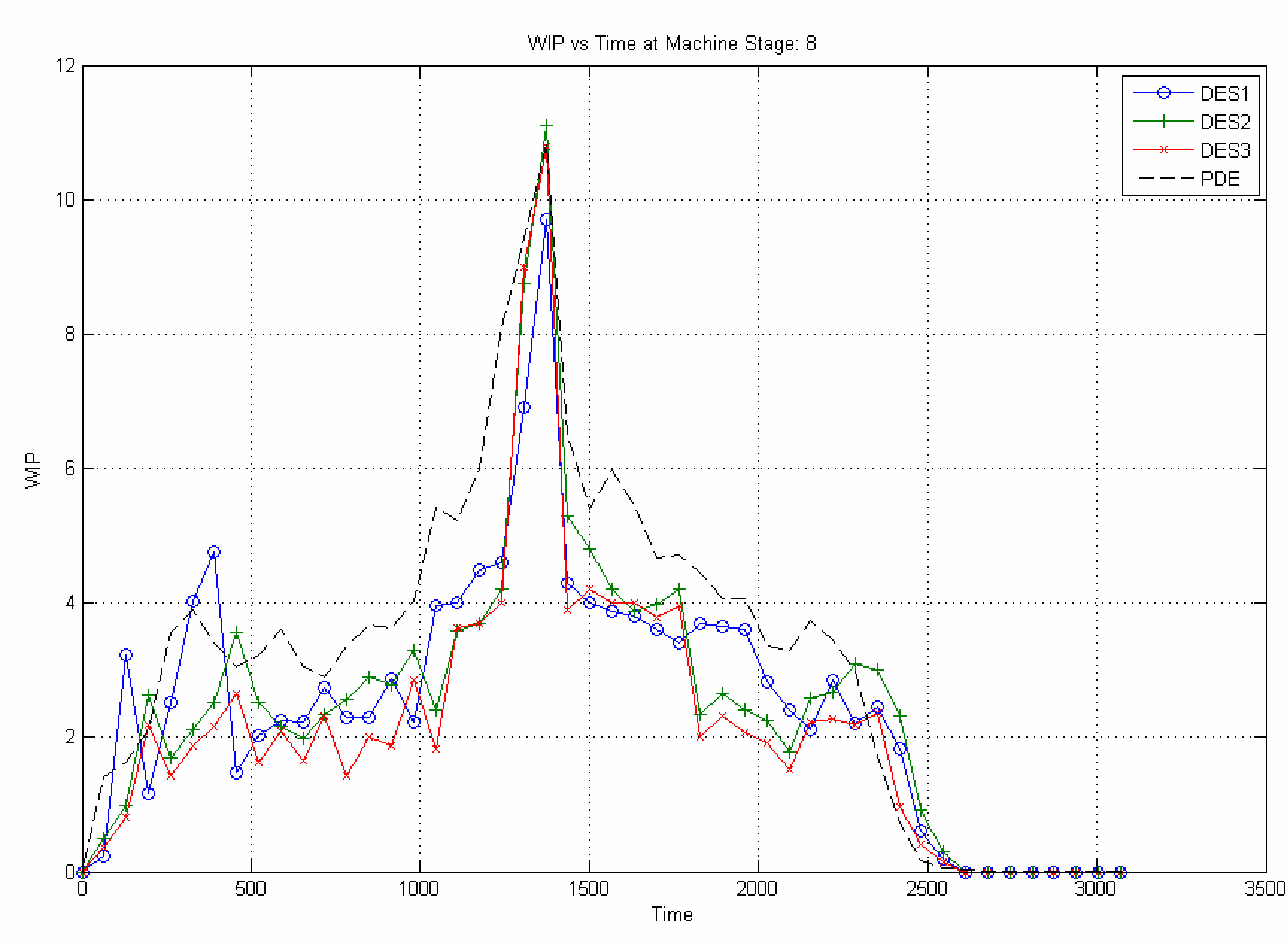


Comparison of Fluxes in 3D






WIP vs Time at Machine Stage: 23
WIP
Time
DES1
DES2
DES3
PDE
0
5
10
15
20
25
30
0
500
1000
1500
2000
2500
3000
3500


WIP vs Time at Machine Stage: 8
WIP
Time
DES1
DES2
DES3
PDE
0
2
4
6
8
10
12
0
500
1000
1500
2000
2500
3000
3500

Conclusions

Experiment with several different test cases are done in order to compare a complex system with the PDE model to substantiate the agreement in terms of WIP levels and fluxes as functions of time at each machine stage.

We conclude:

- We are able to predict the transient behavior of the given system with the macroscopic model under conditions different than those used to collect the data.
- PDE models are preferable to simulate large and complex production systems, because they are accurate, fast, and has a very important advantage as they are amenable themselves to optimization and control.