

RED BLOOD CELL MOTION IN BIFURCATING MICROVESSELS

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INTRODUCTION

- This study on red blood cell distribution in the microcirculation gives a better overall understanding of the microcirculation. Insights from the study could help yield estimates of the distribution of oxygen in the microvasculature which can be used to improve medical treatments.
- The goal of this study is to understand the nonuniform partitioning of red blood cells in vessel bifurcations and the dependence of partitioning on vessel size, orientation, and overall blood flux.
- Understanding red blood cell partitioning in small vessels requires consideration of non-continuum behavior of blood.
- Models of individual red blood cell motion and behavior in bifurcations can capture such non-continuum behavior.
- Red blood cell properties including a viscoelastic inextensible membrane and a viscous cytoplasm should be included to accurately describe individual red blood cell motion.
- In this model, each red blood cell is represented individually as a collection of discrete elements whose mechanical properties represent various properties of the cell.

MODEL AIMS

- To create a two-dimensional model that captures as much of the three-dimensional dynamics as possible.
The two-dimensional model represents the cross-sectional shape of a three-dimensional cell in a plane through the center of the cell.
- To use a minimum number of free parameters and choose them so that the model agrees with experiment.
Model parameters were chosen to produce simulated tank-treading motion which agreed with experiment.

MODEL COMPONENTS

- The cell membrane is represented as a chain of straight elements which are hinged at the outer nodal points and connected by inner elements to a central node.
- Each external element consists of an elastic component in parallel with a viscous component.
- Each internal element is a viscous component.
- Each outer node (hinge point) possesses bending elasticity.

GOVERNING EQUATIONS FOR CELL

External elements representing the membrane have viscoelastic behavior governed by

$$\bar{l}_i = k_i (l_i / l_0 - 1) + \mu_m \frac{1}{l_i} \frac{dl_i}{dt}$$

Here l_i is the length of external element i , l_0 is a reference length, k_i is the elastic modulus and μ_m is the viscosity.

Internal elements have viscous resistance governed by

$$T_i = \mu'_m \frac{1}{L_i} \frac{dL_i}{dt}$$

L_i is the length of internal element i and μ'_m is its viscosity.

An internal pressure is defined by

$$p_{int} = k_p (1 - A/A_{ref})$$

where k_p is chosen to be large so that the cell resists area changes.

At each node, all forces and moments from these elements and from external fluid loadings sum to zero.

GOVERNING EQUATIONS FOR FLUID

The suspending medium is a viscous incompressible fluid governed by the steady Stokes flow equations ($Re \ll 1$).

Pressure and velocity fields are expressed as $p(x,y)$ and $u(x,y)$ where $u = (u,v)$. The components of stress are

$$\sigma_{xx} = 2\mu \partial u / \partial x - p, \quad \sigma_{xy} = \mu (\partial v / \partial x + \partial u / \partial y), \quad \sigma_{yy} = 2\mu \partial v / \partial y - p$$

Conservation of momentum gives

$$\partial \sigma_{xx} / \partial x + \partial \sigma_{xy} / \partial y = 0 \quad \text{and} \quad \partial \sigma_{xy} / \partial x + \partial \sigma_{yy} / \partial y = 0.$$

Conservation of mass gives

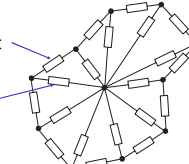
$$\partial u / \partial x + \partial v / \partial y = 0$$

The system of coupled equations for the motion of the cell and the surrounding fluid is solved using a finite element package (FlexPDE), with 20 nodes per red blood cell and 100-500 fluid elements. An explicit trapezoidal method is used for tracking time dynamics.

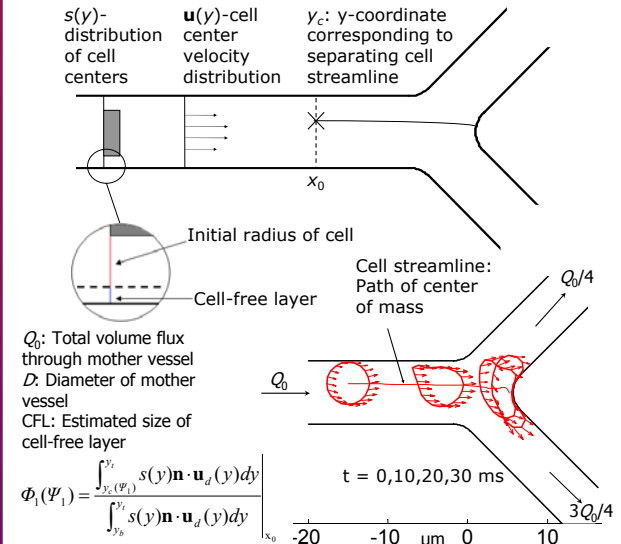
2D MECHANICAL MODEL OF CELL

Viscoelastic element

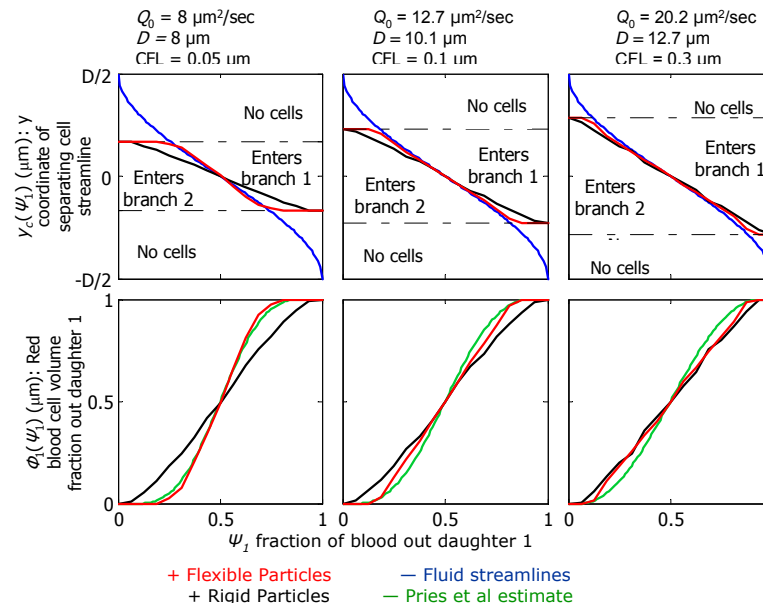
Viscous element



BIFURCATION DETAILS



RESULTS



OUTCOMES

Deviations of red blood cell trajectories from the streamlines of the background flow are caused by:

- Obstruction of daughter vessels
- Flexibility and cell migration tendencies

Flexible cells obstruct daughter vessels less than rigid cells and migrate (rigid cells do not). As such, flexible cells enter high flow branches more than rigid cells.

With appropriate cell-free layers, computational results with flexible particles agree well with experimental results.

PREPRINT

1. Barber, J.O., Alberding, J.P., Restrepo, J.M., Secomb, T.W. Simulated Two-Dimensional Red Blood Cell Motion, Deformation, and Partitioning in Microvessel Bifurcations. Submitted. Available soon:

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