THE VESSEL WALL UNDER THE RAVAGES OF PULSATILE FLOW

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BACKGROUND

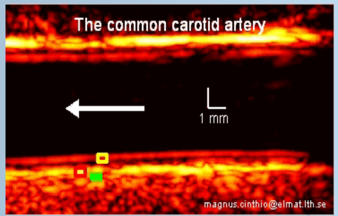
Importance

Shear stress at the vessel wall is suspected to play a role in vascular disease. Atherosclerosis preferentially occurs in areas of disturbed flow or **low shear stress**, whereas regions with normal shear stress seem to be spared.

Previous work:

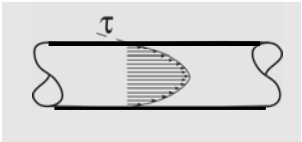
- Emphasis on radial rather than longitudinal displacements.
- Vessel wall treated as thin, having no thickness.
- Wall material treated as rigid or elastic

Recent experimental studies have demonstrated clear longitudinal dynamics within the **thickness** of the vessel wall, [1]



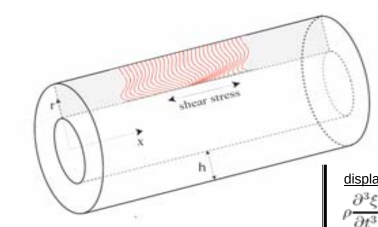
Longitudinal displacements measured within the vessel wall by Cinthio et al , Am J Physiol Heart Circ Physiol, 291: H394, **2006**

PRESENT WORK



- Vessel wall is modeled as having finite thickness.
- Longitudinal displacement caused by shear stress at the vessel wall is used as a boundary condition for an **analytical** solution for longitudinal displacements and stresses **within the thickness of the vessel wall.**
- Wall material is taken as viscoelastic, with variable ratio of viscosity to elasticity to mimic changes with aging or disease.
- Driving forces at the inner boundary are taken as functions of time which can match variations within the cardiac cycle.

MATHEMATICAL PROBLEM



$$\rho\frac{\partial^2\xi}{\partial t^2} = \frac{\partial \sigma_{rx}}{\partial r}$$ - motion equation

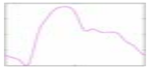
$$\frac{\partial \sigma_{rx}}{\partial t} = E\frac{\partial^2\xi}{\partial t\partial r} - \frac{\omega}{\gamma}\sigma_{rx}$$ - Maxwell model

displacement equation:

$$\rho\frac{\partial^3\xi}{\partial t^3} - E\frac{\partial^3\xi}{\partial t\partial r^2} + \frac{\rho\omega}{\gamma}\frac{\partial^2\xi}{\partial t^2} = 0$$

boundary conditions:

$$\begin{cases} \xi(a,t) = \sum_{n=1}^{10}\left[A_n\cos(n\omega t) + B_n\sin(n\omega t)\right] \\ \xi(a+h,t) = 0 \end{cases}$$



1) Longitudinal displacements within the vessel wall

$$\bar{\xi}(r,t) = \sum_{n=1}^{n=10}\frac{e^{\Lambda_n^{(1)}(1-\bar{r})} - e^{-\Lambda_n^{(1)}(1-\bar{r})}}{e^{\Lambda_n^{(1)}} - e^{-\Lambda_n^{(1)}}}\,\frac{A_n\cos\left(n\omega t - \bar{r}\Lambda_n^{(2)}\right) + B_n\sin\left(n\omega t - \bar{r}\Lambda_n^{(2)}\right)}{\max_t|\xi(a,t)|}$$

$$\bar{r} = (r-a)/h$$

$$\bar{\xi} = \xi/\max_t|\xi(a,t)|$$

$$\Lambda_n^{(1)} = n\omega h\sqrt{\frac{\rho}{2E}\left[\sqrt{1+\frac{1}{n^2\gamma^2}} - 1\right]}$$

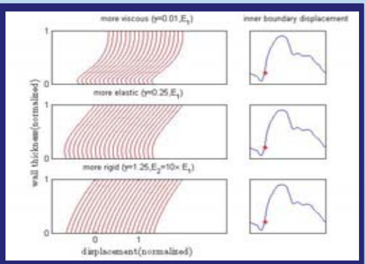
$$\Lambda_n^{(2)} = n\omega h\sqrt{\frac{\rho}{2E}\left[\sqrt{1+\frac{1}{n^2\gamma^2}} + 1\right]}$$

2) Corresponding longitudinal stresses

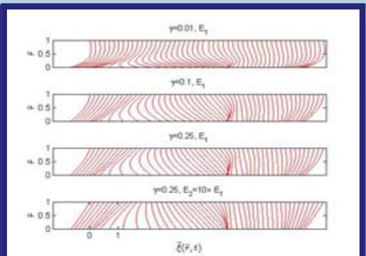
$$\bar{\sigma}_{rx}(\bar{r},t) = \frac{\sigma_{rx}(r,t)}{|\sigma_{rx}(a,t)|}$$

$$= \frac{\sum_{n=1}^{n=10}\frac{1}{\sqrt{1+\frac{1}{n^2\gamma^2}}}A_n Re\left((\Lambda_n^{(1)} - i\Lambda_n^{(2)})e^{\Lambda_n^{(1)}(1-\bar{r})} + (\Lambda_n^{(1)} + i\Lambda_n^{(2)})e^{-\Lambda_n^{(1)}(1-\bar{r})})e^{i(n\omega t - \Lambda_n^{(2)}\bar{r})}\right)}{|\sigma_{rx}(a,t)|} +$$

$$+ \frac{\sum_{n=1}^{n=10}\frac{1}{\sqrt{1+\frac{1}{n^2\gamma^2}}}B_n Im\left((\Lambda_n^{(1)} - i\Lambda_n^{(2)})e^{\Lambda_n^{(1)}(1-\bar{r})} + (\Lambda_n^{(1)} + i\Lambda_n^{(2)})e^{-\Lambda_n^{(1)}(1-\bar{r})})e^{i(n\omega t - \Lambda_n^{(2)}\bar{r})}\right)}{|\sigma_{rx}(a,t)|}$$

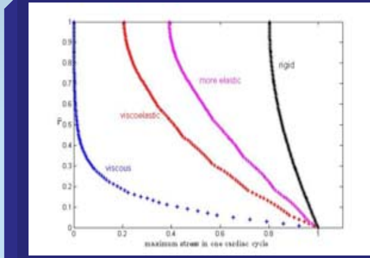


Displacements within the vessel wall for a certain time during systole (shown by star) and for different degrees of viscoelasticity.

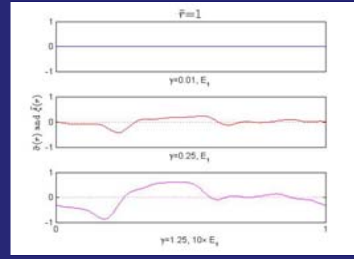


Displacements within the thickness of the vessel wall, during one cardiac cycle and for different degrees of viscoelasticity.

RESULTS



Maximum stress within the thickness of the vessel wall, during one cardiac cycle and for different degrees of viscoelasticity.



Displacements (dotted line) and stresses (solid line) at the outer layer of the vessel wall for different degrees of viscoelasticity and during one cardiac cycle.

CONCLUSIONS

- Oscillatory shear stress or displacement exerted by pulsatile flow on the endothelial layer will be transmitted through the vessel wall in a manner which **depends critically on the degree of viscoelasticity of the wall material.**
- If the wall material is sufficiently viscous, shear stresses and displacements within the wall thickness are confined to a thin layer close to the inner boundary.
- If the wall material is less viscous, more elastic and more rigid, as may occur in disease or aging, the shear stresses are transmitted **almost in full magnitude** from the inner boundary to all layers of the vessel wall.
- **The viscous property of the wall material may thus be seen as a "protective" element** against the axial oscillatory drag force of pulsatile flow, while its absence may mark a deterioration in the mechanical "agility" of the wall.

BIBLIOGRAPHY

[1] M. Cinthio et al. , "Longitudinal movements and resulting shear strain of the arterial wall", AJP-HCP, 291:394-402, 2006

[2] R. Darbey, "Viscoelastic fluids", Marcel Dekker, 1976;

[3] W.W. Nichols & M.F. O'Rouche, "McDonald's blood flow in arteries: theoretical, experimental and clinical principles", 2005

[4] M. Orosz et al., "Viscoelastic behavior of vascular wall simulated by generalized Maxwell models - a comparative study", Med Sci. Monit , 5(3): 549-555, 1999

[5] M.Zamir, "The Physics of pulsatile flow", Springer, New York, 2000