Numerical Simulation of Aggregability of Red Blood Cells in a Micro-vessel by Lagrange Multiplier/Fictitious Domain Method

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Abstract

Aggregation of red blood cells (RBCs) plays a key role in many physiological phenomena. A Distributed Lagrange Multiplier based Fictitious Domain (DLM/FD) method combined with operator splitting techniques and finite element method is developed for simulation of two dimensional motion and aggregation of RBCs in a narrow channel. The cells are modeled as rigid biconcave-shaped neutrally buoyant particles. The force between two cells is modeled by a depletion interaction theory. It is shown through this simulation that the fictitious domain method is applicable to the simulation of aggregability of multiple RBCs in micro vessels.

Introduction

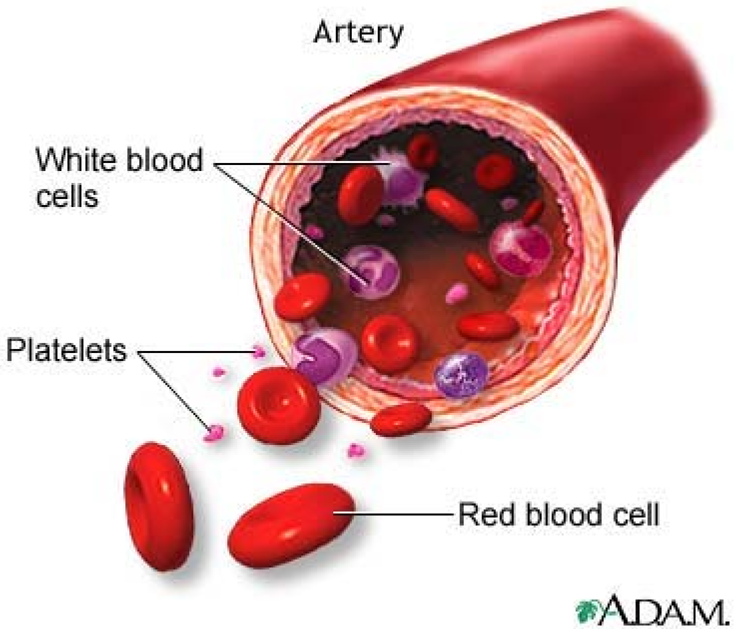
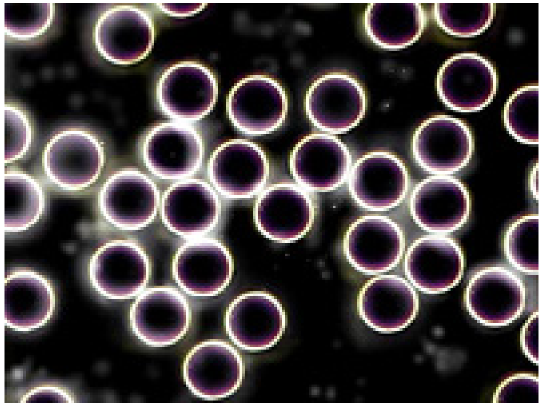


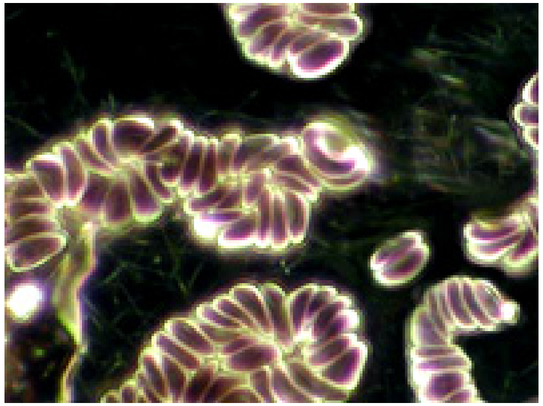
Image is from http://adam.about.com

- **Human red blood cells are biconcave disks;**
- **The diameter of a typical human red blood cell is 6-8 μm;**
- **The fraction occupied by the red blood cells is called the hematocrit. Normally it is approximately 45% ;**
- **The averaged diameter of arterioles is 50 μm, the averaged fluid velocity in arterioles is 5cm/s, the averaged Reynolds number is 0.7.**

Introduction



Healthy live red blood cells



Unhealthy live red blood (Rouleaux are formed which deceases the surface area of the red blood cells and increases the apparent viscosity of the blood. The amount of oxygen and nutrients that can be transported is severely reduced)

Images are from http://www.ginsengup.com/pages/ginseng/germanium.html

Objectives

- To model red blood cells as rigid neutrally buoyant biconcave-shaped particles;
- To model the force between two cells by a depletion interaction model;
- To develop a fictitious domain method for simulating aggregation of red blood cells in a 2-D narrow channel.

Governing Equations

Navier-Stokes equations for the motion of the fluid

$$\rho_f \left[\frac{\partial \mathrm{u}}{\partial t} + (\mathrm{u} \cdot \nabla)\mathrm{u} \right] = \mathrm{F}_e + \nabla \cdot \sigma$$

$$\nabla \cdot \mathrm{u} = 0$$

$$\sigma = -p\mathrm{I} + 2\mu_f \mathrm{D}(\mathrm{u})$$

$$2\mathrm{D}(\mathrm{u}) = \nabla \mathrm{u} + (\nabla \mathrm{u})^t$$

Governing Equations

The Euler-Newton's equations for the motion of cells:

$$v(x,t) = V(t) + \omega(t) \times \overrightarrow{G(t)x}$$

$$\frac{dG}{dt} = V$$

$$M_p \frac{dV}{dt} = M_p g + M_p f_p + F_H$$

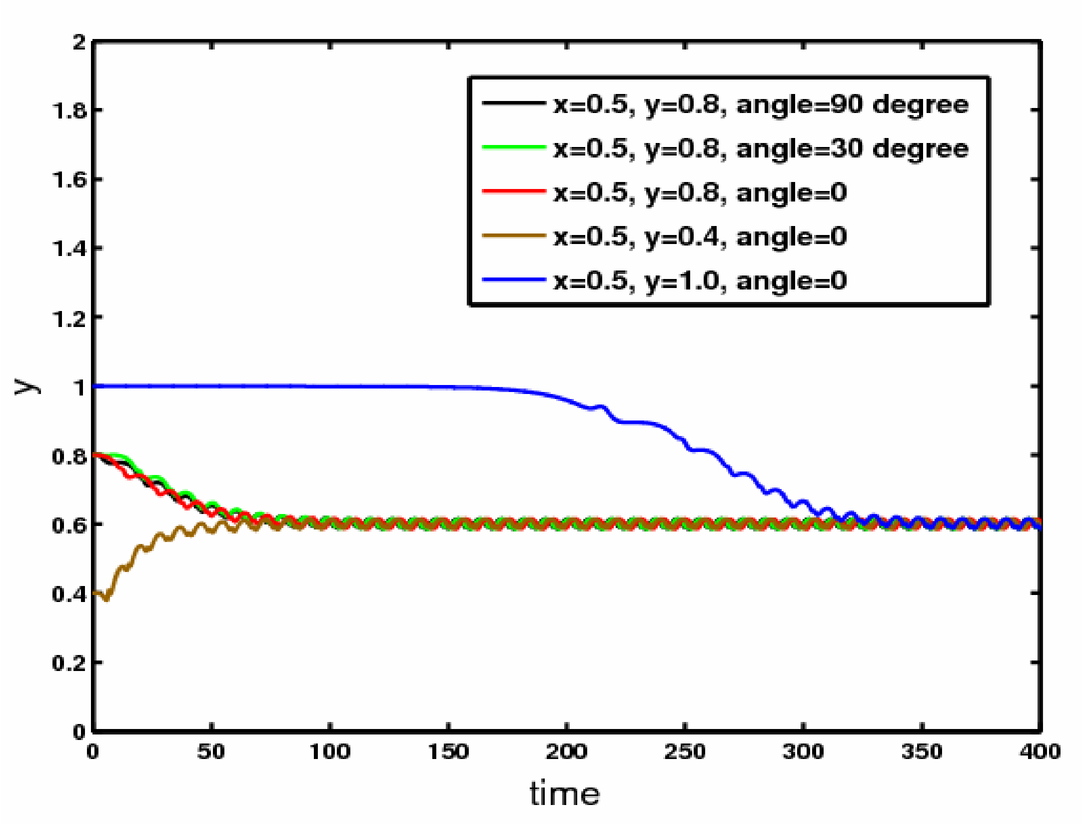
$$\frac{d(I_p \omega)}{dt} = T_H$$

- **V (resp , ω) is the velocity (resp., the angular velocity) of the center of mass G**
- **M_p is the mass of the rigid body**
- **I_p is the inertia tensor of the rigid body**
- **F_H is the resultant of the hydrodynamical forces**
- **T_H is the torque of the hydrodynamical forces acting on the rigid body**

Solution Method

1. Fill the cells with surrounding fluid;
2. Assume that the fluid inside each cell has a rigid body motion;
3. Use 1 and 2 to modify the variational formulation derived from the Navier-Stokes equations;
4. Force the rigid body motion inside each moving cell via a Lagrange multiplier defined (distributed) over the cell;
5. Combine 3 & 4 to derive a variational formulation involving Lagrange multiplier to force the rigid body motion inside the moving cells;
6. Solve 5 by operator splitting techniques and finite element method.

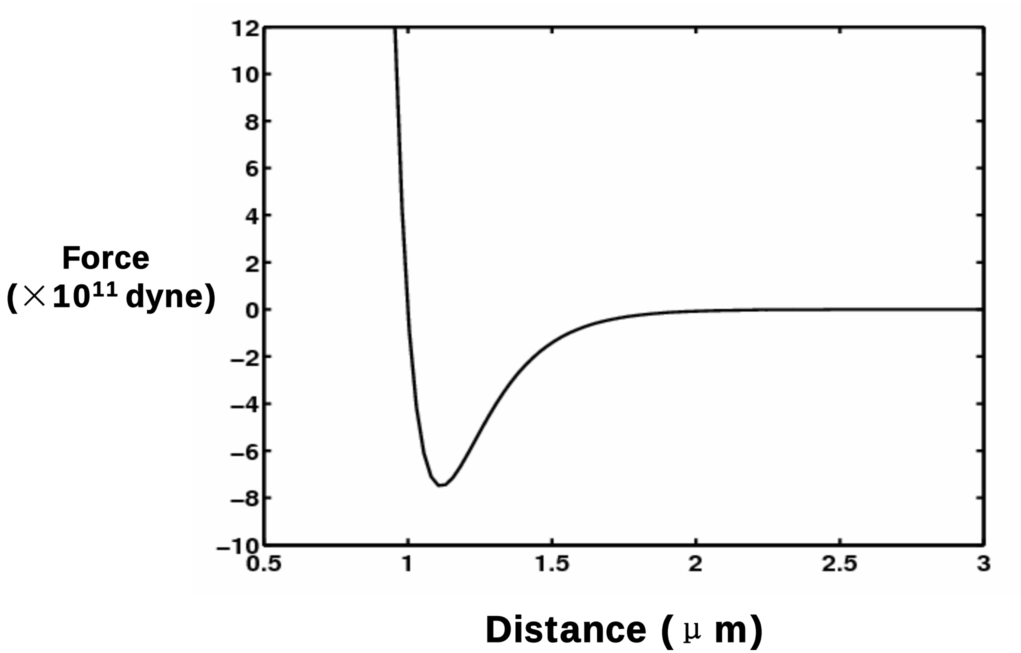
Validation

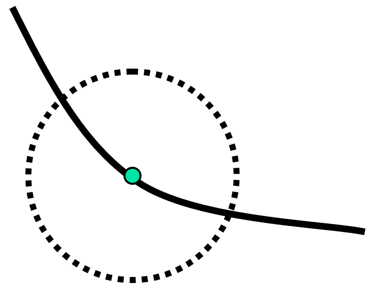


Lateral migration of a biconcave particle in a pressure-driven Poiseuille flow at different initial positions and initial angles. The computational domain is $(0,12)\times(0,2)$, the mesh size for the velocity field is $h_v=1/64$, the mesh size for the pressure is $2h_v$, the particle diameter $d=0.5$, fluid density $\rho=1$, fluid viscosity $\nu=0.1$, time step $\Delta t=0.001$.
At $t=500$, the average particle speed is 1.6277, hence the particle Reynolds number $R_e=8.139$.

Numerical Simulation

- Depletion force $f(r)=2D_e\beta[e^{2\beta(r_0-r)} - e^{\beta(r_0-r)}]$ with surface energy $D_e=5\times10^{-5}\ \mu J/\mu m^2$, scaling factor $\beta=3/\mu m$, zero force length $r_0=1\ \mu m$.





A sphere with the diameter of a cut –off length is used to identify the effective domain of the cell-cell interaction

Numerical Simulation

- Computational domain:

  (0,4)x(0,2) and (0,4)x(0,1.6) ;
- Boundary conditions:

  u=0 on top and bottom wall, periodic in x direction;
- Fluid properties:

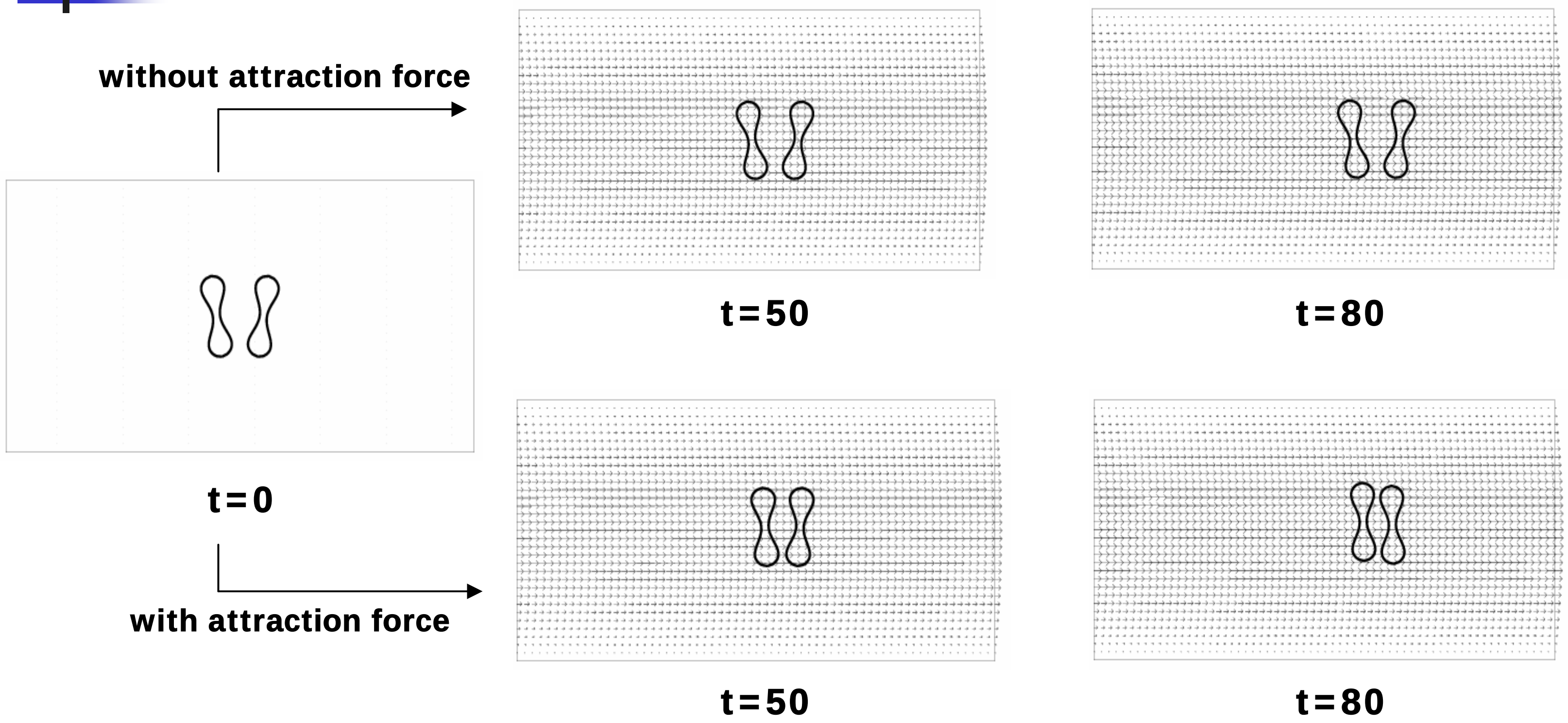
  ν =0.012 g/cm s; $\rho_{fluid} = \rho_{particle} = 1 g/cm^3$
- Shape of red blood cells:

$$\bar{y} = 0.5[1 - \bar{x}^2]^{1/2}(a_0 + a_1 \bar{x}^2 + a_2 \bar{x}^4),$$

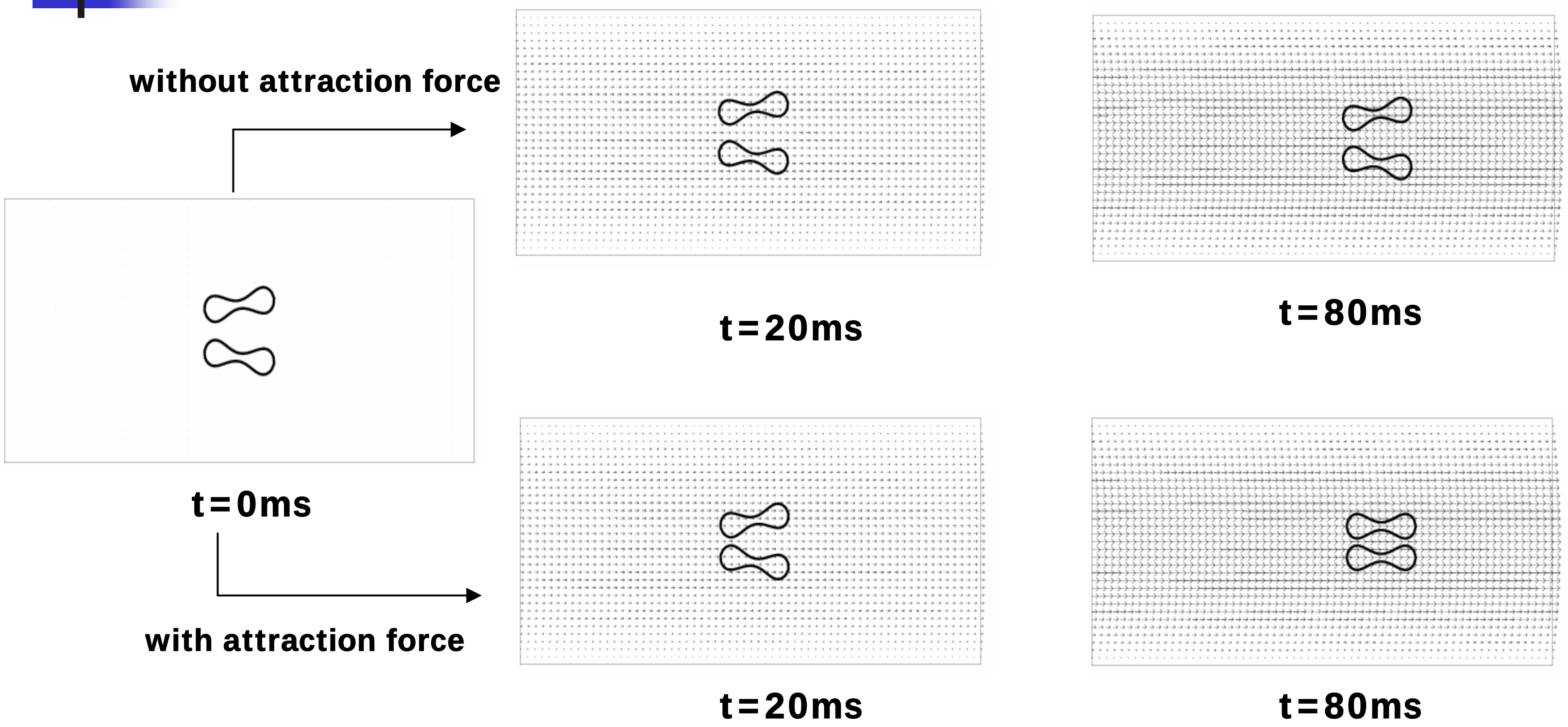
$$with\ \bar{x} = x/d,\ \bar{y} = y/d,\ d\ \text{diameter of the cell}$$

$$and\ a_0 = 0.207, a_1 = 2.002, a_2 = -1.122$$

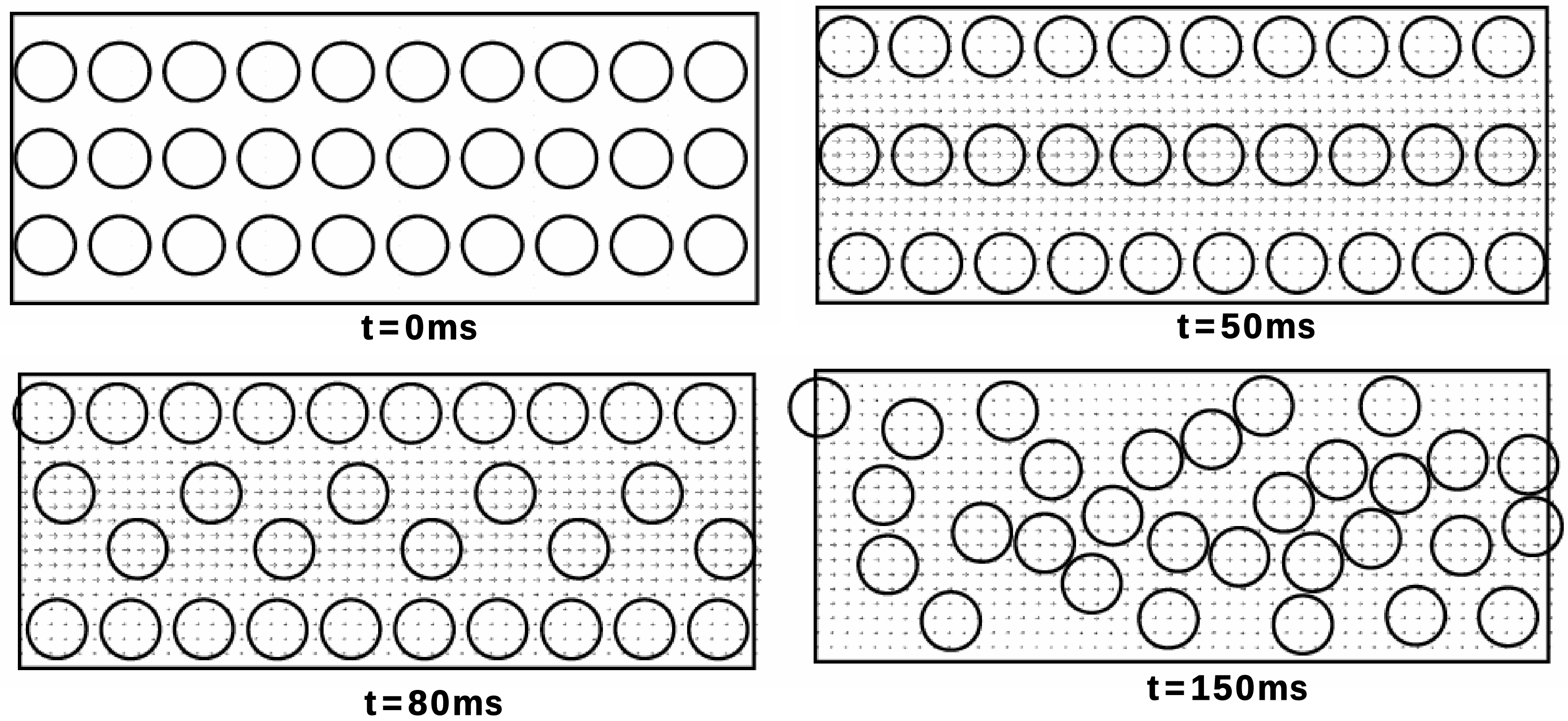
Case 1: comparison of 2 biconcave NB particles with and without attraction force, $De=1\times10^{-5}\ \mu J/\mu m^2$, $\beta=3/\mu m$, $r_0=1\ \mu m$, domain = $(0,40)\times(0,20)\ \mu m$, diameter $d=6\mu m$, maximum fluid velocity at $t=80ms$ is $v=0.09\ cm/s$, $Re=0.01$, the initial angle between two cells is 18 degree.



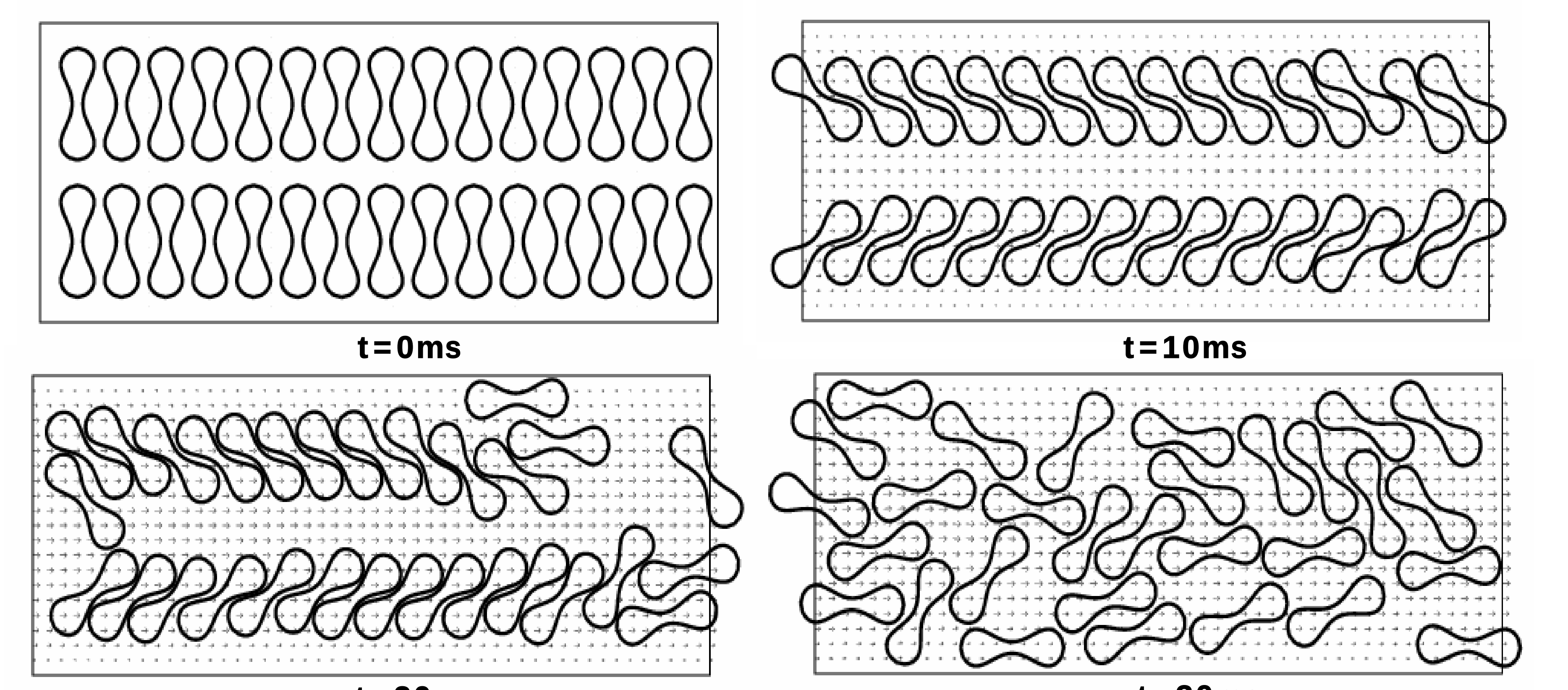
Case 2: comparison of 2 biconcave NB particles with and without attraction force, $De=1\times10\text{-}5\ \mu J/\mu m^2$, $\beta=3/\mu m$, $r_0=1\ \mu m$, domain = $(0,40)\times(0,20)\ \mu m$, diameter $d=6\mu m$, maximum fluid velocity at t=80ms is v =0.09 cm /s, Re=0.01, the initial angle between two cells is 18 degree.



Case 3: 30 sphere NB particles without attraction force, domain = $(0,40)\times(0,16)\ \mu m$, diameter $d=3.2\mu m$, maximum fluid velocity at $t=150ms$ is $v=1.6\ cm/s$, $R_e=0.14$, hematocrit=0.377



Case 4: 30 biconcave NB particles without attraction force, domain = (0,40)×(0,16) μm, diameter d=6μm, maximum fluid velocity at t=30ms is v =3cm /s, R_e=0.201, hematocrit=0.376



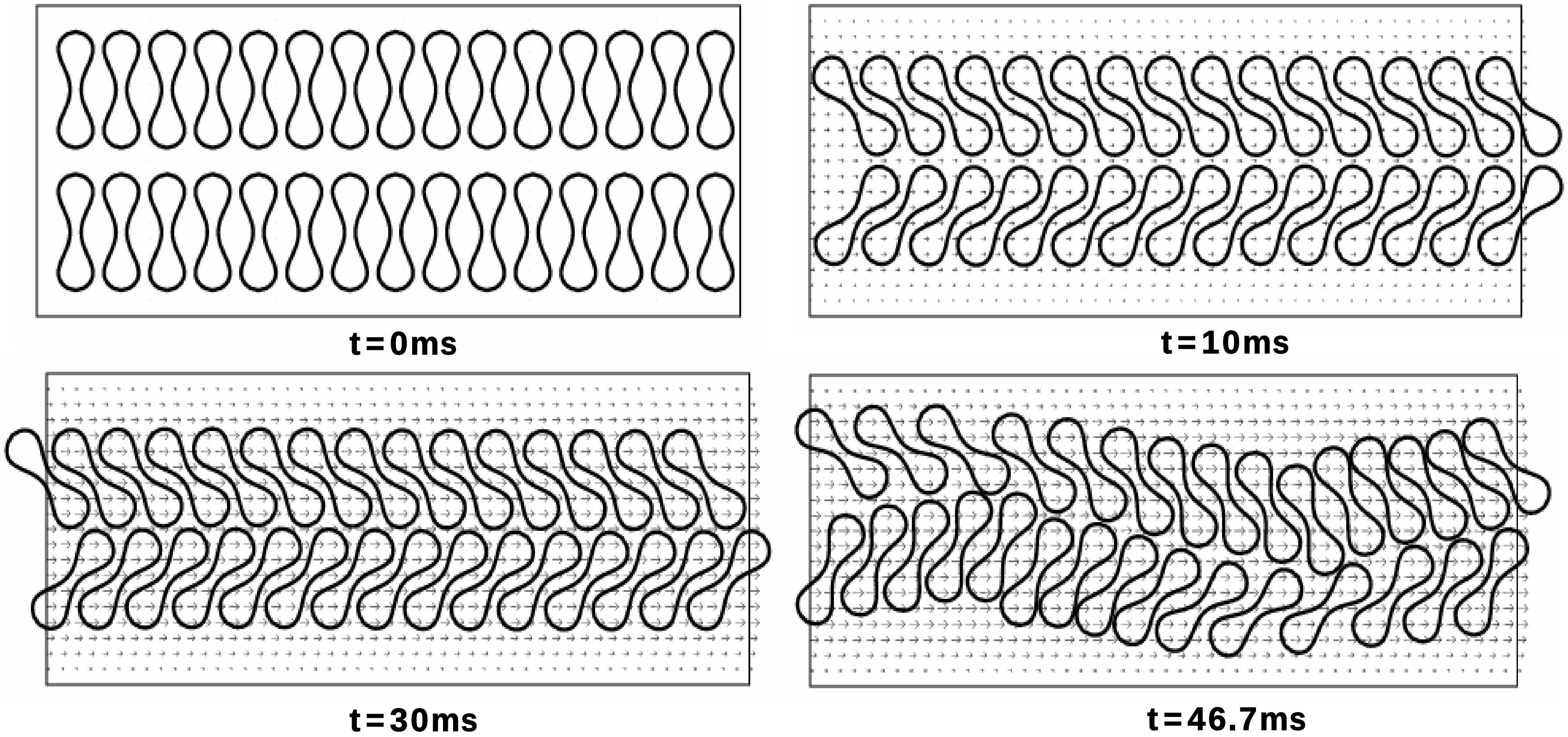
t=0ms

t=10ms

t=20ms

t=30ms

Case 5: 30 biconcave NB particles with attraction force,
$De = 5\times 10\text{-}5\ \mu J/\mu m^2$, $\beta = 3/\mu m$, $r_0 = 1\ \mu m$,
domain = $(0,40)\times(0,16)\ \mu m$, diameter $d = 6\ \mu m$,
maximum fluid velocity at $t = 46.7ms$ is $v = 3\ cm/s$,
$R_e = 0.267$, hematocrit $= 0.376$



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