RIPS 2007
JPL Project Description
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Representation of Families of Invariant Manifolds for Trajectory Design in the Planar Circular Restricted Three Body Problem

Invariant manifolds are crucial to the design of space trajectories. In this project, we want to find a good computational model for families of invariant manifolds in the Planar Circular Restricted Three Body Problem. These manifolds are associated with periodic and quasiperiodic orbits first studied by Poincare in the late 19^{th} century.

We are interested in periodic orbits around the Lagrange points L1 and L2 as well as resonant periodic orbits close to the orbits around L1 and L2. Their manifolds provide a chain for ultra-low-energy transport from one side of the planet to the other. This is the route taken by comets of the Jupiter family like Oterma.

We will use multiple shooting methods to compute periodic orbits. The invariant manifolds will be computed using Floquet theory. We will review the differential geometry of surfaces in R^3 and R^4. Time permitting, we will also study differential forms and some foliation theory. But the ultimate goal is to use these mathematical machinery for computation to produce a tool for handling families of invariant manifolds folded like onion skins or leeks.

One of the tricky points to this representation is that these surfaces actually live in the 3D energy surfaces in the 4D phase space. Thus they are immersed in the 2D configuration space. Moreover, due to chaotic dynamics, pieces of the manifolds are stretched so that they fold and tear in the most violent manner. Clearly we will avoid these regions, and yet they are perhaps the most interesting and most useful portions from scientific and engineering perspectives. Even though we will not spend significant time studying this in detail, we will spend sometime considering how to tackle this beast.

Computations will be done in Matlab. C and Fortran codes are also good, but should be integrated in Matlab to permit interactive analysis. Symbolic manipulations can be done in Mathematica or Maple. A Matlab animation of the final results is highly desirable, but not required. If there is interest, we will also look at using parallel versions of Matlab.

Key Milestones:

1. Computation of unstable periodic orbits in the PRTBP: halo orbits, resonant orbits.
2. Computation of invariant manifolds of unstable periodic orbits.
3. Constructing a multiple shooting tools: a differential corrector. (Best effort basis.)
4. Represent invariant manifold as a surface.
5. Represent families of invariant manifolds.
6. Bonus A: Animation.
7. Presentation at JPL.