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Using Level Sets to Represent Invariant Manifolds to Design a Planet Finder Mission

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Project Description:

Invariant manifolds are crucial to the design of space trajectories. In this project, we want to find a good computational model for families of invariant manifolds in the Planar Circular Restricted Three Body Problem (PCRTBP). These manifolds are associated with periodic and quasiperiodic orbits first studied by Poincare in the late 19^{th} century.

We are interested in periodic orbits around the Lagrange points L2. Their manifolds provide a chain for ultra-low-energy transport from the Earth used by space missions like Genesis. Last year, our team succeeded in computing such surfaces using the level set method. This year we need to extend their work by finding an end-to-end orbit starting from a 200 km circular orbit around the Earth to the periodic orbit around L2 via the invariant manifold surface computed from the level set.

We will use the shooting method to compute periodic orbits. The invariant manifolds will be computed using Floquet theory. Current approach uses specific trajectory segments on the manifold and glue them to the orbit around the Earth using a multiple shooting method. In this project, we will attempt to compute the end-to-end trajectory starting with points on the manifold computed from the level set method. A multiple shooting tool will be provided. The ultimate goal is to use these mathematical machineries to produce a tool for handling families of invariant manifolds folded like onion skins or leeks.

One of the tricky points to this representation is that these surfaces actually live in the 3D energy surfaces in the 4D phase space. Thus they are immersed in the 2D configuration space. Moreover, due to chaotic dynamics, pieces of the manifolds are stretched so that they fold and tear in the most violent manner. Clearly we will avoid these regions, and yet they are perhaps the most interesting and most useful portions from scientific and engineering perspectives. Even though we will not spend significant time studying this in detail, we will spend sometime considering how to tackle this beast.

Computations will be done in Matlab. C and Fortran codes (if used) should be integrated in Matlab to permit interactive analysis. A Matlab animation of the results is required.

Key Milestones:

1. Computation of unstable periodic orbits in the PCRTBP: lyapunov orbits.
2. Computation of invariant manifolds of lyapunov orbits.
3. Represent invariant manifold as a surface using level set algorithms.
4. Find end-to-end trajectory from 200 km orbit around Earth to the Lyapunov Orbit using points on the surface of the level sets computed in step 4.
5. Animation.
6. Presentation at JPL.

References for Level Set Methods:

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S.J. Osher, J.A. Sethian. Fronts propagating with curvature dependent speed. Algorithms based on Hamilton-Jacobi formulations. J. Comp. Physics, 79, 1988, 12-49.

References for Astrodynamics:

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