SPATIAL AND TEMPORARY SIMULATIONS OF ATMOSPHERIC PHASES DISTORTIONS

E.A. PLUZHNIK

Special Astrophysical Observatory , N. Arkhyz, Karachai-Cherkesia, 369167 Russia
Institute of Astronomy of Kharkov University, Sumskaya 35, Kharkov, 61022 Ukraine
e-mail: pea@sao.ru

Abstract

A method for simulations of atmospherically corrupted wavefronts is proposed. The method and the possibility of building both spatial and temporal models of the atmosphere are discussed. The method was used for modeling the Kolmogorov's turbulent atmosphere without Taylor's assumption of "frozen" atmospheric distortions.

1. Introduction

Although the theory of optical waves propagations in the turbulent atmosphere is well developed now, some parts of it still occasionally investigated, mainly experimentally. Particularly this is related to temporal properties of atmospherically corrupted wavefronts. The influence of the corrupted wavefronts on optical imaging systems is an important aspect of understanding the performance of these systems. That is why, development of atmospheric simulation procedures is still relevant.

The most direct way for simulations is spatial filtering for white noise, proposed by McGlamery (McGlamery 1976). But this approach has some limitations due to low-frequency inadequate samples. The methods of simulations based on wavefronts approximations with Zernike polynomials or Karhunen-Loeve functions, as well as the methods based on random generators of processes with the known covariance matrix have been described in details (Roddier 1990, Roggemann et al. 1995, Peterson & Mozurkewich 2003). In this paper we present a new method for simulations both spatial and temporal atmospheric wavefronts variations based on Kolmogorov turbulent model without "frozen" atmosphere assumption.

2. Simulations of spatial variations

The Weiner spectrum $\Phi(k)$ of the phase fluctuations due to Kolmogorov turbulence is given by (Noll 1976)

$$\Phi(k) = 0.023\, k^{-11/3}\, r_0^{-5/3}, \qquad (1)$$

where k is the spatial frequency, and r_0 is Fried parameter. And the phase structure function

$$D_\varphi(r') = 4\pi^2 \int_0^\infty \Phi(k) k \, [1 - J_0(2\pi k r')] dk, \qquad (2)$$

where J_0 is Bessel function. Using the characteristic size $l = 1/k$ of inhomo-

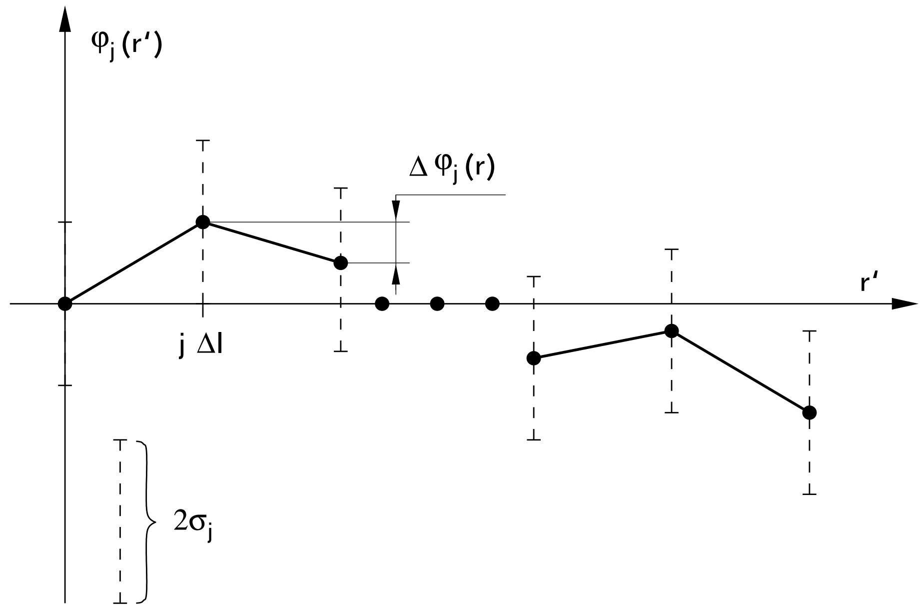

Figure 1. *Generation of a function $\varphi_j(r')$. An increment $\Delta\varphi_j$ has a Gaussian distribution, the mean-square deviation of which is equal to σ_j and the mean value is equal 0.*

geneities, we find that

$$D_\varphi(r') = 0.092\,\pi^2 r_0^{-5/3} \int_0^\infty l^{2/3}\,[1 - J_0(2\pi r'/l)]dl. \qquad (3)$$

Changing the integral in (3) to integral sum yields

$$D_\varphi(r') = 0.092\,\pi^2(\Delta l/r_0)^{5/3} \sum_{j=1}^{\infty} j^{2/3}\,[1 - J_0(2\pi r'/(j\Delta l))]. \qquad (4)$$

The equation (4) only means, that the random process to describe atmospheric phase fluctuations may be present as a sum of random processes $\varphi_j(r')$ associated with inhomogeneous size $j\Delta l$. Structure functions of the processes $\varphi_j(r')$ are equal to

$$D_\varphi^j(r') = 0.092\,\pi^2(\Delta l/r_0)^{5/3} j^{2/3}\,[1 - J_0(2\pi r'/(j\Delta l))]. \qquad (5)$$

Following algorithm can be used to simulate a one-dimensional function $\varphi_j(r')$, supposing that $\varphi_j(r')$ values are correlated for points spaced-apart less than $j\Delta l$ in distance and uncorrelated for other points. The phase increment $\Delta\varphi_j$ for points spaced-apart with $j\Delta l$ distance have the Gaussian distribution, dispersion of which is equal

$$\sigma_j^2 = 0.092\,\pi^2(\Delta l/r_0)^{5/3} j^{2/3}\,[1 - J_0(2\pi)]. \qquad (6)$$

Let $\varphi_j(0) = 0$, then

$$\varphi_j(r') = \varphi_j(\Delta l(n-1)) + \varepsilon\Delta\varphi_j, \qquad (7)$$

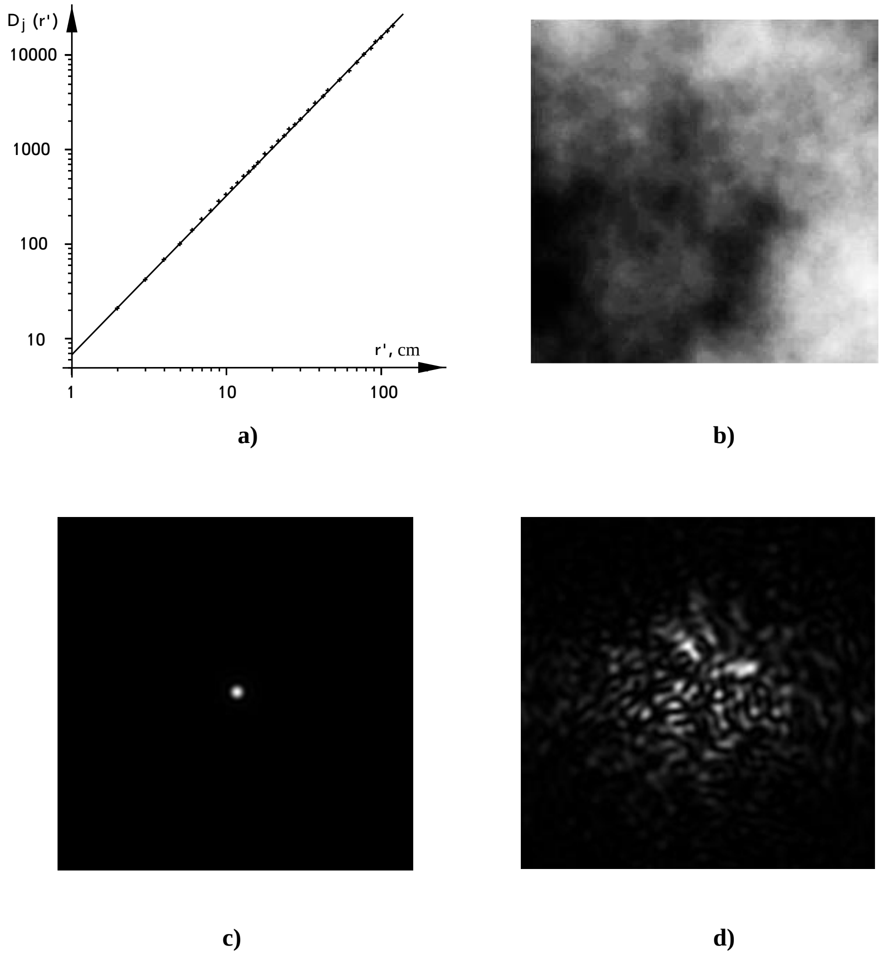

Figure 2. *Simulation of spatial variations of atmosphere wavefronts.*

where

$$\varepsilon = \sqrt{\frac{1 - J_0(2\pi[r' - \Delta l(n-1)]/(j\Delta l))}{1 - J_0(2\pi)}},$$

n is natural number, and $r' \in [\Delta l(n-1), \Delta l\ n]$ (Figure 1). The structure function of the sum

$$\varphi^{(\alpha)}(r') = \sum_{j=1}^{J} \varphi_j(r'), \tag{8}$$

is approximately equal to the theoretical phase structure function

$$D_\varphi(r') = 6.88(r'/r_0)^{\frac{5}{3}}. \tag{9}$$

In equation 8 $J\Delta l$ is the outer scale of turbulence. To create two-dimensional phase distributions we should generate a set of one-dimensional functions $\varphi^{(\alpha)}$, where α is a set of angles, which have the uniform distribution between 0 and 2π, and build two-dimensional phase distribution as

$$\varphi(x', y') = \sum_{\alpha} \varphi^{(\alpha)}(x' \cos\alpha + y' \sin\alpha). \tag{10}$$

The theoretical (line) and model (crosses) structure functions for a set of two-dimensional phase distributions ($r_0 = 1$cm) are presented on Figure 2(a) together with an example of phase distribution generated using the above method (Figure 2,b). The diffraction limited PSF and a model of instantaneous PSF ($D/r_0 = 20$) are also shown on the Figure 2(c,d).

3. Temporal variations

The above approach to atmosphere wavefronts generations make it possible build both spatial and temporal models. Let motions at spatial scale L have a characteristic velocity V. Taking into account Kolmogorov low, we may estimate timescales T_L of spatial inhomogeneities, whose spatial scale is equal L, as (Roddier 1981),

$$T_L \propto L/V \propto \varepsilon_0^{-\frac{1}{3}} L^{\frac{2}{3}}, \tag{11}$$

where ε_0 is the rate of energy dissipation. The equation (11) only means, that lifetime τ_L of inhomogeneities with spatial scale L is proportional to $L^{\frac{2}{3}}$ and depends only on L and the viscosity of air.

Suppose that correlation coefficient $R_\varphi^L(\tau)$ between atmosphere phases due to inhomogeneities with spatial scale L, taken at the moments t and $t + \tau$ at a point on a telescope pupil, can be expressed as

$$R_\varphi^L(\tau) = \begin{cases} 1 - |\tau|/\tau_L & |\tau| \le \tau_L \\ 0 & |\tau| > \tau_L. \end{cases} \tag{12}$$

This model assumes that equal number of inhomogeneities with spatial scale L disintegrate during the same time intervals and all inhomogeneities with spatial scale L are completely disintegrating through time interval $\tau_L = C_t L^{\frac{2}{3}}$, where we set a timescale of temporal phases variations using a constant C_t.

In this way we can expand the above method of spatial atmosphere phases simulations to the case of temporal variations. Let $\Delta\varphi_j(n\Delta\tau)$ is the phase

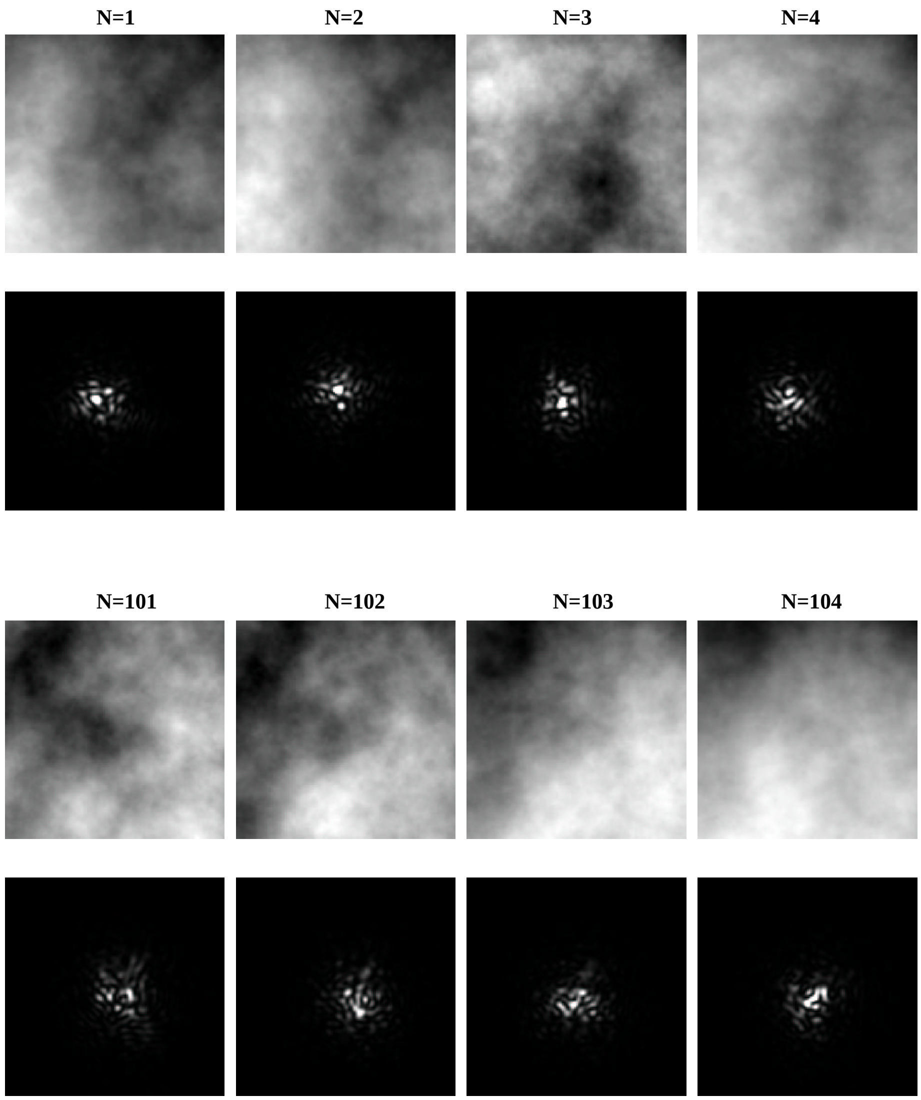

Figure 3. *An example of time evolution of atmosphere wavefronts and associated point spread functions ($D/r_0 = 8$). Realizations numbers are shown.*

increment for inhomogeneities with spatial scale $j\Delta l$ at the time moment $n\Delta\tau$ (look to equation (7)). To calculate wavefront at this time moment, we should generate time set of correlated phase increments $\Delta\varphi_j(i\Delta\tau)$ for each natural number i range from 1 to n. These values have a Gaussian distribution with the covariance matrix **C**, which can be written in according to equation (12) as

$$C[i,k] = \begin{cases} \sigma_j^2(1 - |i-k|/i_{max}) & |i-k| \leq i_{max} \\ 0 & |i-k| > i_{max}, \end{cases} \tag{13}$$

where $i_{max} = C_t j^{\frac{2}{3}} \Delta l^{\frac{2}{3}} / \Delta\tau$. The problem of calculating a set of variables consistent with the covariance matrix **C** has been described by Roggemann et al. (1995) and Peterson & Mozurkewich (2003).

As an example, a set of time varying atmosphere phase screens is shown on Figure 3 together with associated point spread functions ($D/r_0 = 8$).

4. Summary

A method of simulating atmospherical phases screens have been proposed. The method was tested by comparing with well known theoretical Kholmogorov turbulent model. The method includes building of spatial-temporal models, which can be developed for either testing and studying different optical astronomical layout or atmospherical wave propagation. Note that, at present, complete spatial-temporal models of turbulent atmosphere are not exist yet. Proposed algorithms is a way to obtain a simple example of such model. Composing the method with simplified methods (Peterson & Mozurkewich 2003) gives appropriate way to study astronomical imaginary systems, including adaptive optics and long-base interferometers.

Acknowledgements. I am grateful to Prof. D. Peterson of Stony Brook University, Prof. A. Ghez of UCLA, and Dr. M. Al-Wardat (Jordan) for their support of this work.

References

McGlamery B.L., 1976,Proc. SPIE, 74, pp. 225–233.

Noll R., 1976, J. Opt. Soc. Am., 55, pp. 1427–1435.

Peterson, D. M., and Mozurkewich, D., 2003, Proc. SPIE, 4838, pp. 815-823.

Roddier F., 1981, In: Progress in Optics, v. XIX, pp. 283–376.

Roddier N., 1990, Proc. SPIE, 1237, pp. 668–679.

Roggemann M.C., Welsh B.M., Montera D., and Rhoadarmer T.A., 1995, Applied Optics, 34(20), pp. 4037–4057.