Fast Generation and Tracking of GPS Dilution of Precision Regions Using Level Sets

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Project Description: Research in Industrial Projects for Students (RIPS) Program - 2013

(Institute for Pure & Applied Mathematics – IPAM)

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Project Description:

The Aerospace Corporation supports the Air Force's management of the Expandable 24-Satellite Constellation that contains the Global Positioning System's (GPS) primary satellites. The effort to maximize GPS coverage includes the evaluation of several different performance-based metrics that directly relate to the geometry of in-view satellites relative to each user. When evaluating coverage, GPS analysts consider all potential satellite failures because constellation management (and sustainment) efforts must provide high assurance that sudden failures will not cause operational interruptions. GPS performance studies include high precision positioning, high assurance flight and maritime safety, and navigation in difficult terrain (terrain-masking of satellites). Also, GPS studies require the evaluation of all combinations of multiple failures over a fine grid covering the earth and a sidereal day. Even with utilization of a cluster of multi-node computers, this effort requires (at its heart) very efficient calculations of geometry-based performance metrics and their derivatives. Current efficient algorithms developed at Aerospace have focused on grid evaluations and are CPU-intensive. New metrics and analysis capabilities need to be developed to address new receivers that use all satellites in-view.

The development of DOP (dilution-of-precision) displays using Level Set Methods (LSMs) would be of great benefit to the GPS Directorate and members of the GPS User Community. Level Set Methods are numerical techniques for tracking interfaces, curves, shapes, and surfaces on a fixed cartisian grid without the need to parameterize these objects. The potential for a dramatic reduction in processing time is especially important in constellation design, optimization, and maintenance applications where repeated evaluations are necessary in an iterative process (e.g., the number and variety of failures for a thirty satellite constellation or in a combined GPS / Galileo constellation). LSMs have become thriving techniques in the field of image sciences. It is the objective of this IR&D Agile Project to pursue similar benefits in the application of Level Set Methods to manage space-based geopositioning systems and geolocation technologies.

Methods for efficient computation of coverage performance metrics are important for evaluating and optimizing GPS satellite orbits and evaluating user coverage. This project will help to maintain The Aerospace Corporation's expertise in coverage evaluation and constellation optimization techniques.

Work to be Done and Deliverables:

The proposed study for FY13 includes:

1. Familiarize and document:
   a. The fundamental equations of Level Set Methods (LSM).
   b. The GPS dilution-of-precision (DOP) concept.
2. Document:
   a. The project statement.
   b. The project approach.
   c. The project work breakdown structure, assignments, and schedule.
3. Study, implement, compare, and contrast at least two numerical algorithms for executing the LSM for fast generation and time evolution of GPS constant visibility regions.
4. Apply and characterize performance of algorithm/s in item 3 to fast generation and evolution of Horizontal and Vertical DOPs (HDOP and VDOP).
5. Document the software package developed and used for this study.
6. Write a paper describing the study, results, and recommendations.
7. Deliver the final presentation.

Status of Field:

The Aerospace Corporation has played a lead architectural role in the development and sustainment of the Global Positioning System (GPS). The corporation has been very active in supporting all elements of the GPS, including development and management of its satellites, its ground monitoring and control stations, military user equipment acquisitions, security, and systems engineering. In the 1970's and 1980's, most studies of large satellite constellations focused on uniform symmetrically-arranged constellations, which were expected to provide maximum coverage. During this time, funding uncertainties produced uncertainty in the ultimate size of the GPS constellation, which led to the need to deploy a constellation of satellites that could adapt to different numbers of satellites. Funding uncertainties and uncertainties in satellite lifetimes still contribute to the need to maintain a flexible constellation structure.

In response to this need, The Aerospace Corporation developed a flexible 6-plane constellation structure that can adapt to different numbers of satellites and still provide robust coverage equivalent or exceeding that obtainable with less flexible uniform constellation structures. At the heart of The Aerospace Corporation's success in this effort was the development of efficient algorithms for computing geometry-based performance metrics and their derivatives that allowed Aerospace engineers to maximize GPS coverage even in the event of satellite failures.

As user equipment technology continues to improve, applications evolve, and demand for GPS services continue to mount, The Aerospace Corporation has continued to remain the chief architect of GPS and has continued to evolve its constellation analysis capabilities to address the increased demand and support new applications.

References:

1. Parkinson, B.W. and Spilker, J.J., Global Positioning System: Theory and Applications. Volume 1, AIAA, 1996.
2. Massatt, P.D. and Rudnick, K.H. "Geometrical Formulas for the Dilution of Precision Calculation," Navigation, Winter 1990-1991, pp. 379-391.
3. Tsai, R. and Osher, S. "Level Set Methods and Their Application in Image Science," Communications in Mathematical Sciences, Volume 1, Number 4 (2003), 1-20.

Prerequisites:

Required: Linear algebra, Numerical analysis;

Desired: Programming skills (e.g. C language, Matlab)

Keywords:

Global Positioning System, Dilution of Precision (DOP), Satellite Coverage, and Satellite Visibility, Level Set Methods, Constellation Management

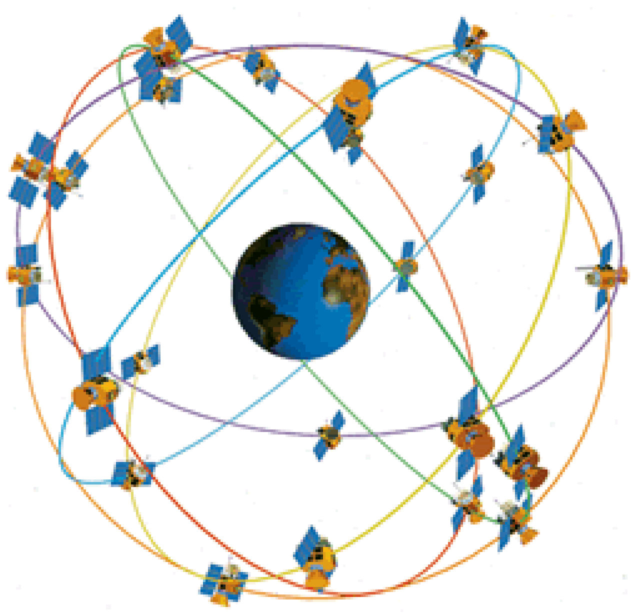

Figure 1. The GPS Constellation, which was originally deployed between 1989 and 1994, is structured as a 27-satellite 6-plane constellation with semi-synchronous (i.e., two periods per sidereal day) circular orbits.

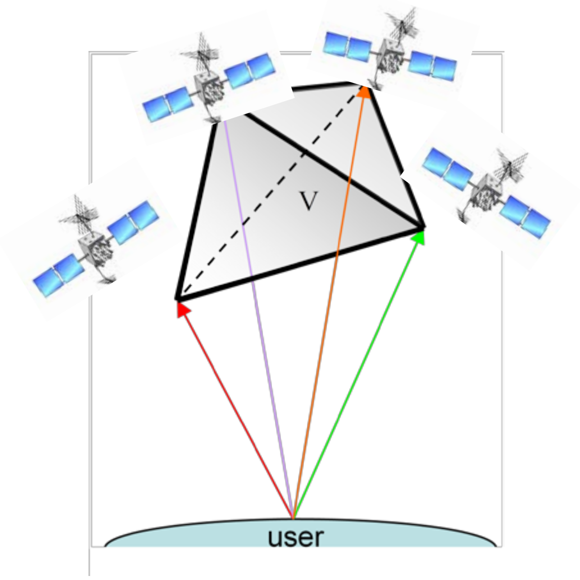

Figure 2. The geometry used in the calculations for DOP. V is the volume of the tetrahedron, which is formed by connecting the endpoints of four unit vectors pointing from a GPS receiver to each satellite.

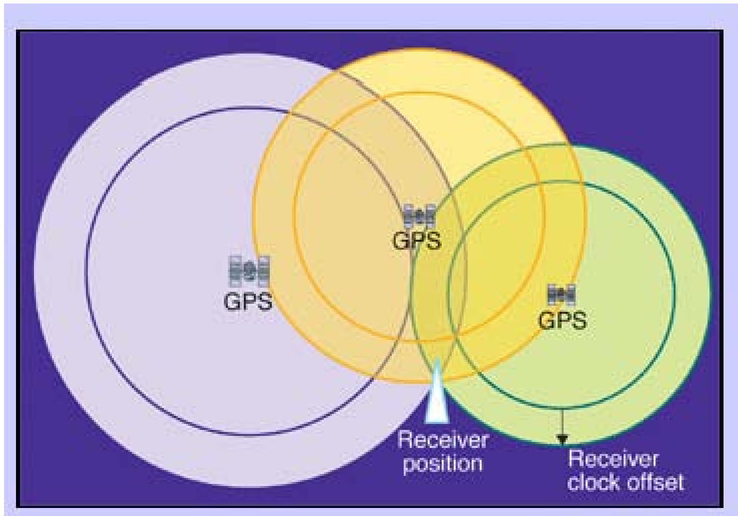

Figure 3. The position of the GPS receiver is where the ranges from a set of satellites intersect at a single measurement time. The range measurements are used together with satellite position estimates based on the precise orbital elements broadcast by each satellite.