New Directions in Mathematical Approaches for Traffic Flow Management

A white paper by participants (see Appendix) of the culminating workshop at the long program
New Directions in Mathematical Approaches for Traffic Flow Management
Institute for Pure and Applied Mathematics, UCLA

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Introduction

Transportation research is currently at a crucial stage: we are facing the emergence of new transformative technologies and systems such as vehicle connectivity, automation, shared-mobility, and advanced sensing which are rapidly changing mobility and accessibility. This in turn will fundamentally transform how transportation planning and operations should be conducted to enable smart and connected communities. On one hand, this process presents us with a great opportunity to build safer, more efficient, reliable, accessible, and sustainable transportation systems. On the other hand, the uncertainties regarding how such disruptive technologies will evolve and how people will react to them pose a number of fundamental challenges. These challenges include: (a) understanding the impacts of connected and automated vehicles on the traffic flow; (b) shifts in travel demand induced by new paradigms in mobility, such as shared mobility; (c) the computational challenges of real-time control strategies for large-scale networks, enabled by emergent technologies; (d) transitioning to predictive and proactive traffic management and control, thus substantially expanding the horizons of transportation network management. The need to effectively address these challenges provides the opportunity for fundamental advances in transportation. Doing so will require a broad and highly interdisciplinary effort among mathematicians, traffic engineers, optimization/control theorists, and computer scientists, amongst others. The NSF-funded Institute for Pure and Applied Mathematics (IPAM) in its 2015 long program on New Directions in Mathematical Approaches for Traffic Flow Management provided a great environment for researchers from these fields to collaborate and take important steps towards a cross-disciplinary effort to addressing these emerging grand challenges in transportation. The white paper was written by the participants of the culminating workshop in December 2015, to provide a comprehensive perspective on the opportunities, the research innovations, the grand challenges, and the needs to enable the community to successfully address them.
Opportunities and challenges

The ongoing and future technological advancements in vehicle connectivity, automation, shared mobility, and data availability are changing our transportation system in fundamental ways.

Connectivity. Connected (but not necessarily automated) vehicles provide information dissemination among different entities in the transportation infrastructure, including vehicles and the built environment. This enables applications that were not possible before. In particular, connectivity facilitates a better understanding of the network performance and it provides additional mechanisms to share information and improve decision-making. The increased availability of information on the state of the network also allows for studying transportation problems in a more holistic and systematic fashion than it was possible in the past. Beyond the primary benefits connectivity provides to enable a wide range of individual and collaborative safety applications, dynamic information and actuation mechanisms also enable novel schemes for routing, pricing/incentives, and infrastructure-based traffic control. Overall, it is expected that both planning and operations would significantly benefit from the development of connectivity, as the basis for numerous automation functionalities described below.

Automation. Automated vehicles have the potential to enhance safety and increase throughput by automating driving-related tasks. The automated tasks, as compared to ones performed by a human, can potentially be processed faster and more accurately using a larger set of information. Perhaps the most important aspect of automation is the creation of additional safety mechanisms such as assisted braking and collision avoidance. However, the possibilities that automation brings go further: it opens the door for complex control strategies to be embedded into the vehicles. For example, automated vehicles can take trajectories that dampen stop-and-go waves and increase the energy-efficiency of other (non-automated) vehicles that will remain on the roadways for many years during the transition to a fully autonomous fleet. A fully automated vehicle fleet also presents opportunities for network control strategies that favor system optimal (SO) routing as opposed to non-cooperative routing (often modeled as a user equilibrium (UE) at an aggregated level).

Together, connected and automated vehicles present a number of crucial challenges that must be overcome before the above-mentioned benefits may be achieved. There are significant uncertainties regarding how these technologies will evolve, and how society will react to them. This can profoundly impact the future car ownership, travel choices, travel demand, and transportation energy use. Ultimately, automation might drastically impact landuse too. The ultimate choice of residence is intrinsically linked to the value of time, which changes under an automated driving paradigm where commute time can be repurposed for work or other activates. It is critical to understand these impacts and develop innovative methods to limit the negative effect of the technologies, while amplifying their positive effect. Ultimately, the introduction of advanced technologies brings new dimensions to modeling, which in turn requires new mathematical tools and paradigms.
Shared mobility. The recent growth of shared-mobility services such as bike sharing (e.g., CitiBike in New York City, Hubway in Boston), car sharing (e.g., Zipcar, car2go), ridesharing (e.g., Uber, Lyft), and ride-pooling (e.g., Uberpool, LyftLine, Via, Bridj) is starting to change the transportation landscape in urban areas. The fundamental difference between shared mobility and individual vehicle ownership is that people can now move independently from their vehicles, leading to more flexibility in travel choices and the potential for more multi-modal trips. This also opens opportunities for promoting more sustainable mobility options such as mass-transit and other ride-pooling services that reduce the number of single-occupancy vehicle trips. Dynamically matching vehicles to riders and routing vehicles to pick up passengers en-route are mathematically difficult problems. New mathematical models are needed to fully understand the emerging problems that come with these shared travel modes. These include the impact on waiting time for a shared vehicle, a possible increase on vehicle miles traveled (VMT) due to passenger-free trips when a vehicle is en-route for a pickup, and a variety of equity issues (e.g., network mobility companies may only operate in areas with high demand and perhaps at the expense of the mass-transit services in these areas, without providing services in sparsely populated areas which are less profitable). Given that the area of shared mobility services is highly dynamic and rapidly evolving, there is an opportunity now to study the long-term impacts of these emerging services and to better manage the mobility infrastructure in cities.

Data availability. The availability of high fidelity data with a broad spatial coverage allows the identification of important transportation phenomena at scales ranging from individual vehicle dynamics all the way to large-scale regional mobility patterns. It also enables the development of corresponding statistical and mathematical models, the validation of these models, and the implementation of network level control and management. Finer-resolution and contextual data allow us to measure, interpret, and predict the response of people to changes in the transportation system more accurately. For example, contextual data streams can reveal system behaviors such as mobility patterns during emergency situations (i.e., via New York City taxi and Twitter data) that were not previously available. While cheap sensing technologies and open data movements have opened up access to data, it is important to be aware of the potential negative impacts that may arise, beyond the obvious security challenges. An important challenge associated with the large influx of data is the scalability of data processing, estimation, and control algorithms that may utilize these data. The data analytics community has devoted a substantial amount of research activity to develop data processing and computational tools (e.g., the Spark framework) that can handle large data volumes. While these tools cannot always be applied in a plug-and-play manner, there is great potential for exploiting these tools to solve large-scale transportation problems that require large amounts of offline and/or streaming data. It is also important to recognize the fact that value of data is highly application driven, even in the backdrop of ever increasing data volumes (e.g., the era of big data). For example, while a moderate penetration rate of GPS equipped vehicles (~1% of total flow) is a great resource for travel-time prediction, it may be insufficient quality data to accurately estimate densities needed for many traffic control applications. The type and quality of the data matters in addition to the amount of data available.

A concerning trend in the transportation community is the growing fragmentation of the data collection, management, and control environments. For example, there are an increased number
of entities that have mechanisms to control urban traffic beyond the traditional municipal authorities. While cities still manage the traffic signal control infrastructure, there are multiple navigation services that can directly influence the traffic flow through the routes they provide. This introduces new control challenges because these entities do not necessarily coordinate their actions with the municipalities or with each due to practical, technical, or competitive reasons. Access to data may also become more challenging, as the entities collecting the vast datasets transition from government agencies to service providers (e.g., ride sharing, connected/autonomous vehicle manufacturers, cellular tower network data). Other issues such as cyber security and privacy need to be considered as an integral part of data collection, modeling, and management (e.g., “privacy by design”), but are not discussed in more detail in this paper.

**Fundamental research innovations**

**Traffic modeling.** Emerging technologies (such as autonomous vehicles entering the roadways) may fundamentally change the way traffic flows, thus potentially requiring the research community to revise existing traffic models. At the same time, emerging technologies can serve as new high-resolution data sources, which enable us to address more challenging research questions, for example: the accurate modeling of (historically difficult to measure) phenomena such as dynamic traffic waves may enable new methodologies to localize high crash risk areas and increased emissions. This information can then be used for proactive traffic control and/or traffic control with environmental considerations. The investigation of these aspects requires the development of new mathematical models as well as the transition of more complex mathematical models into engineering practice.

Examples of such models include: (a) second-order macroscopic models to describe traffic waves and phantom jams; (b) non-local macroscopic models to capture phenomena caused by communicating vehicles; (c) multi-modal and multi-commodity models to describe new mobility patterns; and (d) network models that capture how the various traffic streams interact and evolve under connectivity and automation throughout the network. The mathematical analysis of such new models can serve to identify potential unexpected, non-intuitive pitfalls. For example, if shared information is not used properly, communicating vehicles could actually increase traffic oscillations at a large scale. Mathematical modeling can provide a fundamental, qualitative understanding of new patterns in traffic flow, before actual data about it is available.

To transition new traffic models to practice (including real-time control), more sophisticated computational methods for macroscopic models will be required. This includes new numerical schemes that effectively resolve non-local interactions, and high-resolution schemes to help localize high-risk areas such as shocks.

**Real-time estimation, prediction, and control.** The new types of data that have become available recently, or will become in the near future, such as various types of in-vehicle sensors, will enable us to conduct real-time estimation and forecasting of the traffic state with fidelity that
was impossible with traditional sensor data. The accurate and highly resolved modern sensor data, when combined with appropriate state-estimation techniques applied to advanced models for traffic dynamics and travel demand, can provide predictions of traffic states into the future. Such forecasting approaches can enable advancements such as: (a) optimal rerouting advice, even for long trips; (b) control of dynamic speed limits and ramp-meters; (c) control of future connected autonomous vehicles to prevent localized congestion peaks; (d) proactive traffic management, (e.g., to smooth traffic flows in areas with high accident risk); (e) and optimal on-demand availability of shared vehicle fleets.

In addition to fundamentally predictive control tasks, many control strategies that operate in real time also require a predictive component to operate effectively in the presence of delays due to communication and computation time. Moreover, real-time traffic estimates are a critical service in the detection and response to incidents such as traffic accidents. While traditional services driven by sensor data have proven to not exceed the detection time of human messaging, modern data streams and connected vehicle technologies have the potential to substantially lower the detection time and simultaneously activate mitigating control actions (e.g., rerouting, corridor signal control, and emergency dispatch).

When moving from real-time estimation and control to prediction, uncertainty quantification and ensemble computations become crucial (as no measurement data will be available to correct for errors in the models, parameters, demands, or initial state). Nonintrusive sampling-based techniques (e.g., Monte-Carlo) are structurally simple but may become impractical due to their high computational cost. In contrast, intrusive techniques (e.g., polynomial chaos) can address these issues. In addition, proper adaptive and robust control strategies must be devised to process probabilistic state predictions. Real-time and predictive control also imposes fundamental computational challenges due to the size and complexity of the transportation infrastructure when combined with large data volumes and communication delays. An interdisciplinary community effort (including computer scientists and mathematicians) is needed to enhance the expertise in relevant areas such as uncertainty quantification, parallel computing, domain decomposition, load balancing, and distributed computing for problems facing our transportation systems. These areas are critically needed to scale the computational toolset required to study transportation problems at ever increasing scales and real-time constraints.

**Network effects.** State-of-the-art traffic assignment models integrate traffic dynamics based on first-order nonlinear partial differential equation (PDE) models such as the Lighthill-Whitham-Richards (LWR) model, but they have some limitations in terms of their scalability and computation time due to the complexity of the underlying assignment problem. With the inclusion of connected and autonomous vehicles, the assignment problems will need to incorporate even more complicated traffic models (e.g., to capture the drivers’ reactions to these vehicles), which will further increase the complexity of these problems. Moreover, new numerical methods may be needed to address flow allocation problems at the junctions in the network. These problems contain mathematically challenging existence and uniqueness problems with regards to the solution of the PDE in certain cases, such as when the multi-commodity route demands are time-
varying. Furthermore, the well-posedness of the underlying PDE models has important implications for the computational methods used for engineering applications.

Network effects are best understood with improved demand data. For years, demand models have relied on slowly evolving data sources such as census data, which at best can represent trends but is not able to capture dynamic features of mobility. When complemented with mobile phone data now available (e.g., Call Data Records), it is very likely that demand models will experience a rebirth over the next decade, leading to much improved network effect models capable of characterizing correlations, linkages, and coupling at large scales.

Another emergent network level problem arises from the high penetration rate of navigation services by multiple route providers that allow real-time traffic rerouting during accidents or unusual traffic conditions. This may lead to undesired consequences resulting from a lack of coordination between these providers, such as congestion of arterial roads onto which traffic has been diverted. These effects could be mitigated by coordination, but the competitive landscape for the individual providers (which are mostly commercial entities), renders this a challenging problem.

One possibility for network-level traffic management is to introduce adaptive or time-dependent pricing and incentive schemes that work to push the system toward a system-optimum equilibrium. While congestion pricing has been successfully implemented in some cities, historically it has been difficult to test or deploy, in part because of the need for expensive physical infrastructure to enforce the pricing. With the advent of connected vehicles, this infrastructure may no longer be necessary, and deployment of congestion pricing or incentive schemes will be greatly simplified. In order to formulate such schemes, there are fundamental mathematical questions to be addressed on calculating the appropriate dynamic pricing strategies and innovative demand management strategies. Furthermore, the pricing strategies can be even more complex when only a fraction of the vehicles are subjected to the incentive or toll (i.e., Stackelberg games), due to only partial compliance or limited participation in an incentive mechanism.

Data for model validation. The development of new mathematical models (including network coupling conditions at highway ramps) requires high-fidelity traffic data that allows one to conclude precise car-following and acceleration profiles. Such trajectory data that includes every vehicle on the roadway are extremely rare. One widely used example is the NGSIM data set, which has enabled many advances in model development and validation. However, it faces fundamental limitations: the road segment is very short; the time intervals are brief; the reported acceleration information is not accurate; and important effects (lane-changing, ramp dynamics, carpool lane) are strongly intermixed. These restrictions limit its use for the validation of modern traffic models, such as second-order macroscopic models, in which acceleration plays an important role.

To address this crucial shortcoming, researchers that came together at the IPAM long-term program aim to produce a new data set that maintains the advantages of the NGSIM data set, while remedying its drawbacks. Specifically, considering a longer highway segment for a longer
duration in time, thus capturing the evolution of traffic patterns such as stop-and-go waves; accurate acceleration information; the inclusion of more than one ramp; and in turn containing a segment that is not strongly affected by ramp effects. The goal of this community effort is to include researchers from all relevant disciplines, ranging from mathematicians working on model development, transportation engineers and data scientists, to experts on the sensor development and deployment.

The collection of high-fidelity data of real traffic flow must be complemented by traffic experiments that single out and carefully study specific aspects of traffic flow, including: the dynamics of stop-and-go waves in the presence of mixed-human-and-automated-vehicle traffic; or effective traffic light control at intersections affected by vehicle-infrastructure communication. In particular, experiments enable the use of instrumented vehicles to collect more accurate acceleration data than the observation of real traffic flow would allow.

With new high-quality trajectory and experimental data leading to more accurate traffic models, better estimation and control approaches are induced as well. For example, it can enable a better treatment of situations where the control directly affects microscopic aspects (e.g., ramp-metering induces lane-changing) that are not captured in a macroscopic model. Here, the trajectory data can provide a fundamental understanding these microscopic phenomena.

Enabling mechanisms

Several mechanisms are needed to enable the community to address the fundamental research challenges identified above. In particular, we need to extend and foster collaboration across disciplines, and develop mechanisms to make data collection, experimentation, research, and education more efficient and straightforward.

Building and expanding interdisciplinary communities. Addressing the new challenges in transportation requires an increasingly broad and cross-disciplinary effort. Several distinct research communities must engage in collaborative activities. However, there is a need for more and improved mechanisms to enable and encourage the exchange of knowledge, expertise, and tools. Improved collaboration mechanisms will lower the barrier to new interdisciplinary activities, increase the shared knowledge-base throughout the community, and improve awareness of existing resources to support research investigations.

Institutes such as IPAM at UCLA play an extremely important role in bridging the mathematical and engineering communities. In addition to community building activities such as the long program New Directions in Mathematical Approaches for Traffic Flow Management in Fall 2015, a sustained effort of similar activities is needed to have maximum impact on the above mentioned research challenges. To further facilitate the exchange across disciplines, important trends and emerging topics can be highlighted in special issues targeted at research innovations from both mathematical and engineering perspectives. Other practical aspects include the development of a common exchange where communities can share knowledge, data, code, experiments, and
best practices. It is envisioned that a community-driven portal of knowledge delivery and sharing can expedite and encourage participation of researchers and engagement with the community.

Collaboration. To address the challenges of the rapidly advancing transportation landscape, there is an urgent need to grow and sustain the interactions and synergies among researchers across the relevant disciplines. Many research groups study similar problems in modeling, traffic estimation, traffic control, and dynamic traffic assignment from different angles ranging from theoretical investigations to engineering practice. Tools, events, and activities to further support a growing interdisciplinary community in transportation research will maximize the impact of activities ranging from experimentation to applied mathematical investigations.

Experimentation. Experimentation is a critical aspect of studying the future needs for emerging technologies such as smart intersections, and connected and autonomous vehicles. These experiments can belong to one of two general categories: one-time experiments, which are conducted over a short time scale to answer very specific questions (for example the 2008 Japan ring road experiment and the extended NGSIM experiment proposed in this paper), and ongoing experiments, which involve the continuous monitoring of a system over a long period of time. To facilitate future ongoing experiments, open-access to dedicated test centers across the country is needed to address specific transportation research questions. These test centers could also partner with public agencies and private industry to gather measurement data from existing devices that is not currently being made available for research (e.g., loop detector data in many municipalities, intersection controller data, etc.). Such data sharing and warehousing could be facilitated through public-private partnerships with universities, experimentation centers, and industrial partners (as appropriate) that are gathering but not yet widely distributing the data in the research community.

Data collection and role of professional societies. Private companies collect a vast amount of data that is potentially relevant for research. However, engaging with private companies to obtain transportation-related data for academic use is always a challenging endeavor due to the different objectives and constraints in industry and academia, despite the potential benefits of such collaboration. In some cases, there have been significant financial costs borne by industry to collect and share the data, and as a consequence the data may have restrictions on what and how the data and subsequent analysis of the data can be disseminated in the broader scientific communality.

One possibility to encourage further productive interaction between academia and industry would be through engagement with professional societies such as TRB, IEEE, and INFORMS. Notwithstanding the remaining technical issues that would need to be addressed, society-sponsored transportation data competitions (where a company may release a dataset for researchers to analyze and submit their results) allow industry high visibility while generating focused scientific effort on solving practical problems. The data could then be made publicly available to support future research.
**Data collection centers.** There is a need to establish large data warehouses, along with data processing, analysis, and sharing tools that can be used by the whole community. Other scientific communities have used this concept successfully. For example, few particle accelerators and high power telescopes exist, but the outcome of experiments conducted at these facilities benefits the whole community.

In the transportation field, there is a need to develop facilities at a variety of scales to support development, use, and sharing. A few large-scale testbeds, similar to MCity (a full scale urban testbed for autonomous and connected vehicles developed at Michigan), may be sufficient if good working mechanisms are established. At smaller scales, facilities may exist at the level of individual faculty research labs at universities, but the need for sharing and information exchange remains high.

Two creative methods can be leveraged to increase participation and collaboration: (a) sharing of space, equipment, and instrumentation time slots as part of collaborative experimentation efforts; (b) sharing of data and data processing tools, formats, applications/techniques, and software. Given the ever-increasing level of expertise required to deal with new technologies and systems, these methods can increase research activities in a large network of centers accessible to all researchers. This will also allow optimal integration and utilization of centers dealing with different topics.

By developing and using additional exchange mechanisms such as a web portal, researchers with specialized expertise can be in charge of maintaining data in a searchable format to encourage and facilitate collaboration, allow data replication, and increase efficient exchange of results. A promising example of an open data exchange in the area of connected vehicles is the Research Data Exchange run through the USDOT Connected Data Systems Program.

**Outreach to agencies and practitioners.** Through tech transfer programs, and professional education programs, it will be possible to bring the practitioners communities closer to the academic communities in a way which will provide greater linkages between the problems faced in the field and the problems researched in academia. This can be done two ways. Academia can contribute to educate the practitioner community by creating the proper outreach classes to teach them the latest in traffic modeling and data analytics. Vice versa, academia can create forums to learn more from practitioners to motivate academic research and steer it towards an application-oriented set of goals (which still will rely on sound and advanced mathematical techniques).

**Education.** The next generation of transportation systems is built upon the advancement of sciences and technologies that enhance transportation safety and efficiency. Ultimately, next generation transportation professionals are the ones who will have to face and address all the above-identified challenges. The required skill set for future transportation professionals is vast, including mathematical modeling, network sciences, data analytics, economics, computer programming and computational skills, information and control theory, among others. Most essential transportation toolboxes live across various disciplines, such as statistics and mathematics, physics, civil engineering, computer sciences, electrical and computer engineering,
industrial engineering and operations research, economics, and the list is growing. Similar to the need to further enhance research and collaboration across boundaries of traditional disciplines (as described above), the transportation community needs to explore new opportunities to teach transportation to attract the broadest set of talent and expertise to the field.
Appendix. List of authors and editors
This white paper was prepared by participants of the culminating workshop of the IPAM long program *New Directions in Mathematical Approaches for Traffic Flow Management* in December, 2015.

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