Perfectoid Quantum Physics and Diamond Nonlocality

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Abstract

We introduce perfectoid quantum mechanics, which is quantum mechanics enriched over perfectoid spaces, diamonds, and 'etale cohomology of diamonds, in the sense of Scholze [5].

We conjecture a correspondence between geometric points in the diamond and entanglement entropy, extended to the effect of nonlocality on entanglement entropy [3], in the diamond setting. Ours is a geometrization of nonlocality in a non-Noetherian complete valuation ring.

Perfectoid Spaces

Definition [1]: A perfectoid space is an adic space covered by affinoid spaces of the form $Spa(R, R^+)$ where R is a perfectoid ring.

Perfectoid Shimura variety [3]:
$S_{K^p} \sim \varprojlim_{K^p} (S_{K^pK_p} \bigotimes_E E_p)^{ad}$.
Lubin-Tate tower at infinite level [2], [6]
$\mathcal{M}_{LT,\infty} = \tilde{U}_x \times^{GL_2(\mathbb{Q}_P)_1} GL_2(\mathbb{Q}_P) \cong \sqcup_Z \tilde{U}_x$.

Any completion of an arithmetically profinite extension [4]; p-divisible formal group laws.

$Spa(K, K^+)$ for K a perfectoid field and $K^+ \subset K$ a ring of integral elements. Zariski closed subsets of an affinoid perfectoid space.

References

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[2]Fargues, L., *Geometrization of Local Correspondence, an Overview*, arXiv:1602.00999 [math.NT].
[3] Caraiani, A. and Scholze, P. *On the generic part of the cohomology of compact unitary Shimura varieties*, Annals of Mathematics (2) 186 (2017), no. 3.
[4] Narain, G. and Zhang, H-Q, *Non-locality effect on the entanglement entropy in deSitter*, arXiv:1812.08667 [hep-th].
[5] Scholze, P., *'Etale Cohomology of Diamonds*, arXiv:1709.07343 [math.AG], 2017.
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Diamond

Definition [6]: Let $Perfd$ be the category of perfectoid spaces and $Perf$ be the subcategory of perfectoid spaces of characteristic p. A **diamond** is a pro-'etale sheaf $\mathcal{D}$ on $Perf$ which can be written as the quotient X/R of a perfectoid space X by a pro-'etale equivalence relation $R \subset X \times X$.

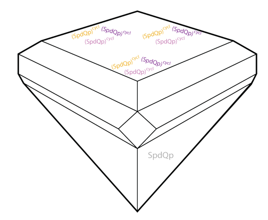

Figure 1: Diamond $SpdQ_p = Spa(Q_p^{cycl})/Z_p^{\times}$ [1]

Diamond Examples

$SpdQ_p = Spa(Q_p^{cycl})/\underline{Z_p^{\times}}$ [6].

$SpdQ_p$ is the coequalizer of

$\underline{Z_p^{\times}} \times Spa(Q_p^{cycl})^{\flat} \rightrightarrows Spa(Q_p^{cycl})^{\flat}$.

$\diamond$ product: $SpdQ_p \times_{\diamond} SpdQ_p$.

Relative Fargues-Fontaine Curve [1]:

$\mathcal{Y}_{S,E}^{\diamond} = S \times (Spa\mathcal{O}_E)^{\diamond}$.

Moduli space of shtukas for $(\mathcal{G}, b, \{\mu_1, ..., \mu_m\})$ fibered over $SpaQ_p \times SpaQ_p ... \times_m SpaQ_p$ [6].

$K^{Efimov}(\mathcal{Y}_{S,E}^{\diamond})$; Efimov K-theory of Diamonds[1]

Let C be an algebraically closed affinoid field and $\mathcal{D}$ a diamond.
A **geometric point** $Spa(C) \rightarrow \mathcal{D}$ is "visible" by pulling it back through a quasi-pro-étale cover $X \rightarrow \mathcal{D}$, resulting in profinitely many copies of $Spa(C)$ [6].

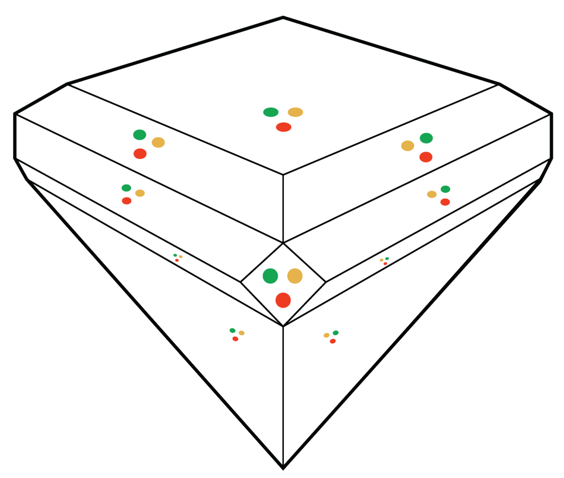

Figure 2: Geometric Point $Spa(C) \rightarrow \mathcal{D}$ [1]

Main Conjectures and Diamond Nonlocality

Dictionary

Quantum Physics	Perfectoid Quantum Physics
Hilbert space	perfectoid space
state vectors	geometric points $Spa(C) \rightarrow \mathcal{D}$
$\bigotimes$ product	$\diamond$ product $SpdQ_p \times_{\diamond} SpdQ_p$
nonlocality	profinitely copies of $Spa(C)$
superposition	pro-'etale sheaves on Perf; profinite sets
wavefunction collapse	tilting; perfectoid modular curves S_{K^p}
entanglement	six functor formalism
quantum topology	'etale cohomology of diamonds
operator algebra	non-Noetherian complete valuation ring
unitarity	pro-'etale descent datum

Conjecture 1: There exists an $(\infty, 1)$ category of diamonds with pro-'etale descent datum.

Remark 1: We are interested in pro-'etale descent datum for unitarity.

Conjecture 2: Geometric points $Spa(C) \rightarrow \mathcal{D}$ in the diamond are a geometrization of entanglement entropy in a non-Noetherian complete valuation ring, taking values in $\mathcal{Y}_{S,E}^{\diamond} = S \times (Spa\mathcal{O}_E)^{\diamond}$.

Remark 2: We are using 'geometerization' in the sense of 'making $Spec(E)$ geometric' in a GAGA correspondence for $\mathcal{Y}_{S,E}^{\diamond} = S \times (Spa\mathcal{O}_E)^{\diamond}$ [1], [6].

Remark 3: The global 'visibility' of the geometric points is in the profinitely many copies of $Spa(C)$. Multiple 'profinitely copies' result from multiple quasi-pro-'etale covers. Perfectoid entropy measures the number of quai-pro-'etale covers.

Remark 4: We propose perfectoid entanglement entropy as a profinite form of 'up to' restricted to the pro-'etale site and to pro-'etale morphisms, which take values in

$\mathcal{Y}_{S,E}^{\diamond} = S \times (Spa\mathcal{O}_E)^{\diamond}$.

Remark 5: The 'up to' takes the form of Scholze's six operations in the 'etale cohomology of diamonds ([5] Def. 1.7iv, Theorem 1.8).
For any map $f : Y \rightarrow X$ of small v-stacks that is compactifiable, representable in locally spatial diamonds and with dim.trg $f < \infty$, a functor $Rf^! : D_{'et}(X, \Lambda) \rightarrow D_{'et}(Y, \Lambda)$ that is right adjoint to $Rf_!$.
Proposition 2.2 [2]. Bun_G is a stack on $Perf$.

Conjecture 2.3 [2]: Bun_G is a "smooth diamond stack".

Conjecture 3A: Nonlocality modularity takes the form $S_{K^p} \sim \varprojlim_{K^p} (S_{K^pK_p} \bigotimes_E E_p)^{ad}$.

Conjecture 3B: Nonlocality is geometrized in the 'etale cohomology of diamonds [5].

Remark 6: Diamond nonlocality is a perfectoid version of nonlocality that arises from the nontrivial geometry of the diamond product $SpdQ_p \times Spd_{Q_P}$. Perfectoid rings are highly non-Noetherian.

Remark 7: Narain and Zhang show that strong non-locality tends to decrease the long-range entanglement in the infrared [3].

The moduli space of shtukas is a diamond fibered over $SpaQ_p \times ... \times_m SpaQ_p$ [3]. We give long-range entanglement the structure of fibering over m-fold products.

Remark 8: Galois Nonlocality as in [2] Theorem 1.3: To any perfectoid field K, associate a perfectoid field K^b of characteristic p, the tilt of C. The absolute Galois groups of K and K^b are isomorphic.