Perfectoid Quantum Physics and Diamond Nonlocality

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Abstract

We introduce perfectoid quantum mechanics, which is quantum mechanics enriched over perfectoid spaces, diamonds, and 'etale cohomology of diamonds, in the sense of Scholze [5].

We conjecture a correspondence between geometric points in the diamond and entanglement entropy, extended to the effect of nonlocality on entanglement entropy [3], in the diamond setting. Ours is a geometrization of nonlocality in a non-Noetherian complete valuation ring.

Perfectoid Spaces

Definition [1]: A perfectoid space is an adic space covered by affinoid spaces of the form Spa(R, R+), where R is a perfectoid ring.

Perfectoid Shimura variety [3]: $K_p \approx \lim(S_{K_p,E} \otimes \mathbb{E}_p)^{\text{et}}$.

Lubin-Tate tower at infinite level [2], [6], $\mathcal{M}_{LT,\infty} = \mathcal{U}_\infty \times \mathcal{G}_L(Q_p) \times \mathcal{G}_L(Q_p) \cong \mathcal{U}_\infty$.

Any completion of an arithmetically profinite extension [4], $p$-divisible formal groups laws.

Spa(K, K+) for K a perfectoid field and K+ ⊂ K a ring of integral elements. Zariski closed subsets of an affinoid perfectoid space.

Diamond

Definition [6]: Let Perf be the category of perfectoid spaces and Perf be the subcategory of perfectoid spaces of characteristic p. A diamond is a pro-etale sheaf $D$ on $\text{Perf}$ which can be written as the quotient $X/R$ of a perfectoid space $X$ by a pro-etale equivalence relation $R \subset X \times X$.

Main Conjectures and Diamond Nonlocality

Quantum Physics

Hilbert space

state vectors

$\otimes$ product

nonlocality

superposition

wavelength collapse

entanglement

quantum topology

operator algebra

unitarity

Perfectoid Quantum Physics

perfection space

geometric points $\text{Spa}(C) \to \mathcal{D}$

$\otimes$ product $\text{Spa}_p \times \text{Spa}_p$

pro-etale sheaves on Perf; profinite sets

tilting; perfectoid modular curves $\mathcal{S}_K$.

six functor formalism

‘etale cohomology of diamonds

non-Noetherian complete valuation ring

pro-etale descent datum

Dictionary

Conjecture 1: There exists an $\langle \infty, 1 \rangle$ category of diamonds with pro-etale descent datum.

Remark 1: We are interested in pro-etale descent datum for unitarity.

Conjecture 2: Geometric points $\text{Spa}(C) \to \mathcal{D}$ in the diamond are a geometrization of entanglement entropy in a non-Noetherian complete valuation ring, taking values in $\mathcal{Y}_{S,E} = S \times (\text{Spa}_E)^{\text{et}}$.

Remark 2: We are using ‘geometrization’ in the sense of ‘making Spec(E) geometric’ in a GAGA correspondence for $\mathcal{Y}_{S,E} = S \times (\text{Spa}_E)^{\text{et}}$.

Remark 3: The global ‘visibility’ of the geometric points is in the profinitely many copies of $\text{Spa}(C)$. Multiple ‘pro-infinitely copies’ result from multiple quasi-pro-etale covers. Perfectoid entropy measures the number of quasi-pro-etale covers.

Remark 4: We propose perfectoid entanglement entropy as a profinite form of ‘up to’ restricted to the pro-etale site and to pro-etale morphisms, which take values in $\mathcal{Y}_{S,E} = S \times (\text{Spa}_E)^{\text{et}}$.

Remark 5: The ‘up to’ takes the form of Scholze’s six operations in the ‘etale cohomology of diamonds [5] Def. 1.7iv, Theorem 1.8.

Conjecture 3B: Nonlocality is geometrized in the ‘etale cohomology of diamonds [5].

Remark 6: Diamond nonlocality is a perfectoid version of nonlocality that arises from the nontrivial geometry of the diamond product $\text{Spa}_p \times \text{Spa}_p$. Perfectoid rings are highly non-Noetherian.

Remark 7: Narain and Zhang show that strong non-locality tends to decrease the long-range entanglement in the infrared [3].

The moduli space of shtukas is a diamond fibered over $\text{Spa}_p \times \text{Spa}_p$. We give long-range entanglement the structure of fibering over m-fold products.

Remark 8: Galois Nonlocality as in [2] Theorem 1.3. To any perfectoid field $K$, associate a perfectoid field $K^0$ of characteristic $p$, the tilt of $C$. The absolute Galois groups of $K$ and $K^0$ are isomorphic.