Learning-based Traffic State Reconstruction using Probe Vehicles

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Why do we need traffic reconstruction?

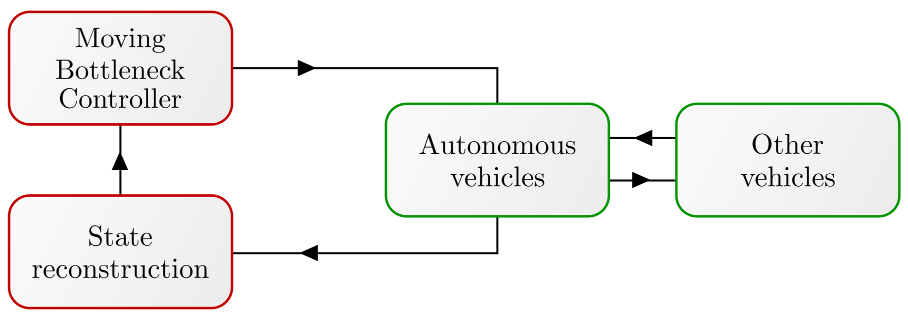
The booming number of city-dwellers impacted the traffic flow by creating jams. Congestion, in turn implies:

- an increase of travel time,
- more emissions,
- an elevated risk of accident,
- stressful situations...

Since it is not always desirable (nor possible!) to change the road infrastructure, the easiest solution is to **control the traffic** using, for instance,

- traffic lights,
- variable speed limits,
- **autonomous vehicles**.

The following block diagram shows how to control the traffic flow using autonomous vehicles and the moving bottleneck strategy.



In the case discussed here, we are interested in reconstructing the density only. It is then more appropriate to consider that we are using **probe vehicles**, capable of sensing the density around them using state-of-the-art sensors.

Traffic flow model

We will use here a macroscopic description of traffic to model the density of vehicles. ρ is the normalized density, meaning that $\rho = 0$ implies that there is no car and $\rho = 1$, the cars are touching each others.

The Lighthill-William-Richards (LWR) equation is the following partial differential equation:

$$\rho_t + (\rho V(\rho))_x = 0 \tag{1}$$

with an appropriate initial condition. V is the instantaneous speed function and, in the LWR model, it is assumed to be:

$$V(\rho) = V_f(1-\rho). \tag{2}$$

However, since the solutions might be discontinuous, another possible dynamic is:

$$\rho_t + (\rho V(\rho))_x = \gamma^2 \rho_{xx}, \tag{3}$$

where $0 < \gamma \ll 1$ is a diffusion correction term.

Since we are using probe vehicles, we need to use a microscopic description of traffic flow as well. The N probe vehicles follow the dynamics

$$\dot{x}_i = V(\rho(x_i)), \qquad i = 1, 2, \dots, N. \tag{4}$$

with appropriate initial conditions. These vehicles are probing, so we measure the following quantity:

$$y(t) = \begin{bmatrix} \rho(t, x_1(t)) \\ \vdots \\ \rho(t, x_N(t)) \end{bmatrix}.$$

What is a reconstructed density?

We want to reconstruct the density ρ on the time-window $[0, T + \Delta T]$ knowing $y(t)$ for $t \in [0, T]$.

> **Definition:** Let $(\mathcal{H}, \|\cdot\|_{\mathcal{H}})$ be a semi-normed vector space, $\mathcal{H}_c \subset \mathcal{H}$ and $\rho \in \mathcal{H}$. We say that $\hat{\rho}$ is a **partial-state reconstruction** of ρ if
>
> $$\hat{\rho} \in \arg\min_{\bar{\rho} \in \mathcal{H}_c} \|\rho - \bar{\rho}\|^2_{\mathcal{H}}. \tag{5}$$

1. A solution of (1) belongs to $\mathcal{H} = H^1(\Omega, [0,1])$ where
$$\Omega = \left\{ (t,x) \in \mathbb{R}^+ \times \mathbb{R} \mid x \in [x_1(t), x_N(t)],\ \forall t \in [0, T + \Delta T] \right\},$$
2. The reconstructed signal $\hat{\rho}$ belongs to
$$\mathcal{H}_c = \{\rho \in C^\infty(\Omega, [0,1]) \mid (3) \text{ holds}\} \subset \mathcal{H}, \tag{6}$$
3. A semi-norm on $\mathcal{H}$ is
$$\|\bar{\rho}\|^2_{\mathcal{H}} = \sum_{i=1}^{N} \int_0^T \bar{\rho}(t, x_i(t))^2 \, dt.$$

How to solve the optimization problem?

The optimization problem (5) is very difficult to solve. One generic solution is to use a neural-network. Traditional neural-network are nevertheless not able to deal with the constraint (6). One solution is to use physics informed deep-learning with the following cost function:

$$\mathcal{J} = \mu \mathcal{J}_{\text{est}} + (1-\mu)\mathcal{J}_{\text{phy}} + \gamma^2$$

where $\mu \in [0,1]$ and

$$\mathcal{J}_{\text{est}} = \frac{1}{N \cdot N_{\text{est}}} \sum_{i=1}^{N} \sum_{j=1}^{N_{\text{est}}} \left| \rho(t_j, x_i(t_j)) - \hat{\rho}(t_j, x_i(t_j)) \right|^2,$$

$$\mathcal{J}_{\text{phy}} = \frac{1}{N_{\text{phy}}} \sum_{j=1}^{N_{\text{phy}}} \left| \hat{\rho}_t(t_j, x_j) + (\hat{\rho} V(\hat{\rho}))_x (t_j, x_j) - \gamma \hat{\rho}_{xx}(t_j, x_j) \right|^2.$$

This is a discretization of the optimization problem (5).

> **Theorem:** if $N_{est}, N_{phy} \to \infty$, the neural-network solution is a partial-state reconstruction.

Finite-difference simulation

We simulate (3) with a small γ and (4) using a finite-difference scheme and a random initial condition. The result is displayed in Figure 1.

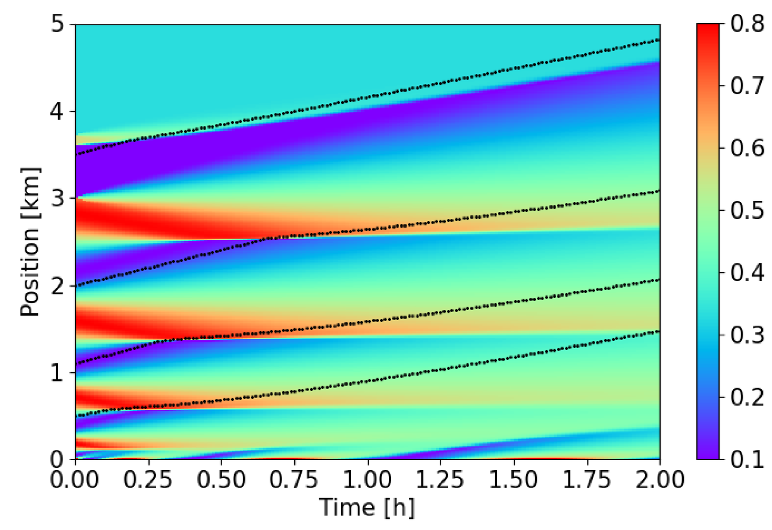


Figure 1: *Finite-difference approximation of the solution of* (3)-(4) *with random initial conditions. The lines refer to the solution of* (4) *and the color scale is related to the density.*

The neural network solution is provided in Figure 2 under noisy measurements. We can note the following:

- There are some discrepancies between the two solutions for small time but they are fading,
- The prevision is accurate,
- The reconstruction is robust to noise.

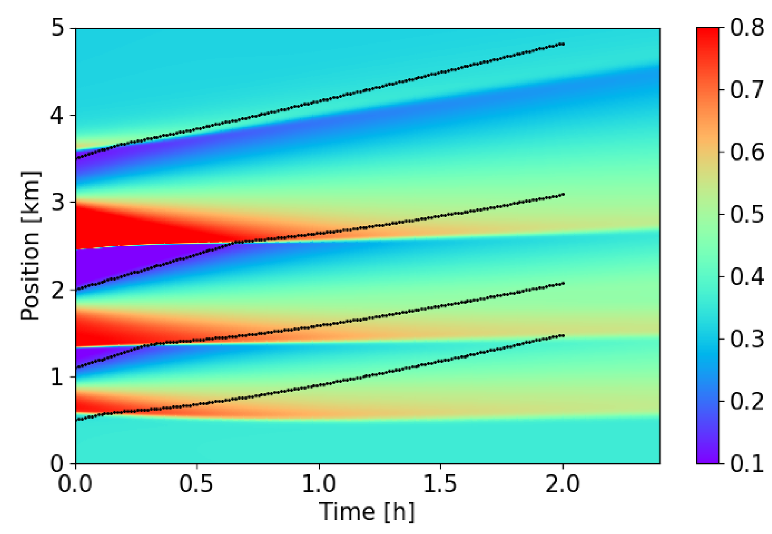


Figure 2: *Neural-network solution of* (5) *(with $\Delta T = 0.5$h). The lines refer to the trajectories of the probe vehicles and the color scale is related to the density.*

SUMO simulation

We want now to study the robustness with respect to model uncertainties. We then use SUMO to get a solution with stop-and-go waves as displayed in Figure 3.

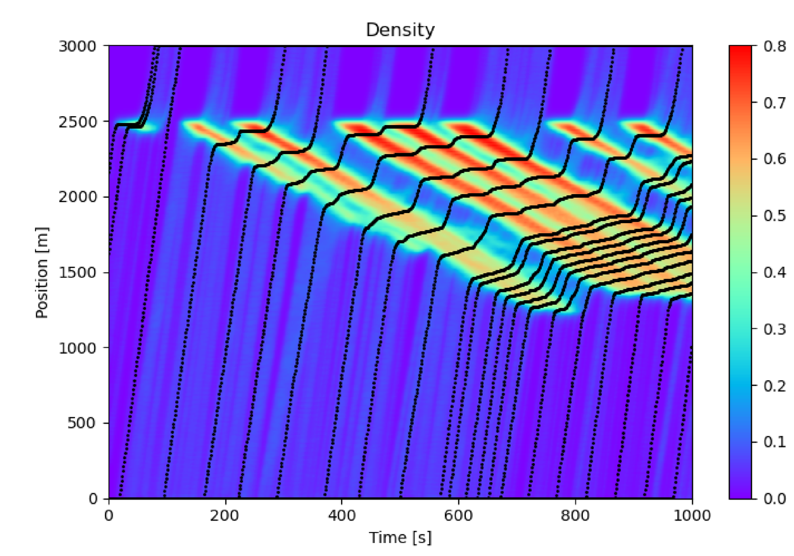


Figure 3: *SUMO simulation. The lines are the trajectories of the probe vehicles and the color refers to the density.*

The velocity function associated to this simulation needs to be calibrated. A regression on the measured velocity of the probe vehicles leads to the identification in Figure 4.

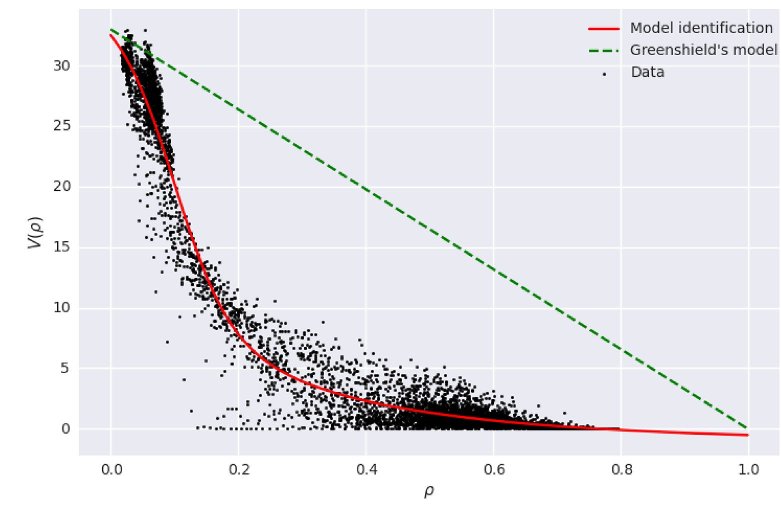


Figure 4: *Velocities measurements, their optimal approximation (red) and the model approximation* (2).

Using this new V function in the optimization problem (5) leads to the neural-network solution plotted in Figure 5,

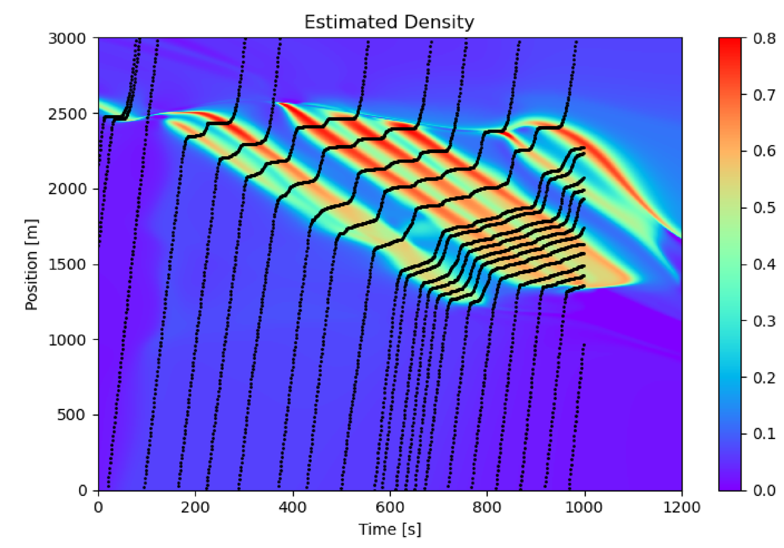


Figure 5: *Neural-network solution of* (5) *(with $\Delta T = 200$s). The lines refer to the trajectories of the probe vehicles and the color scale is related to the density.*

We observe that:

- There are some discrepancies between the two solutions that are **not** fading,
- The prediction is relatively accurate,
- The reconstruction is sensitive to noise.

Future work

This is a preliminary study and it paves the way to a more elaborate methodology. The perspectives are:

1. Move to a second-order model,
2. Formalize and robustify the approach,
3. Improve the identification step.

Acknowledgements

This research is partially funded by the KAUST Office of Sponsored Research under Award No. OSR-2019-CRG8-4033, the Swedish Foundation for Strategic Research and Knut and Alice Wallenberg Foundation. The authors are affiliated with the Wallenberg AI, Autonomous Systems and Software Program (WASP).

References

Liu, J., Barreau, M. and Johansson, K.H. Learning-based Traffic State Reconstruction using Probe Vehicles. *Arxiv*, 2020. Under review.

Barreau, M., Liu, J., and Johansson, K.H. Learning-based state reconstruction of a scalar hyper-bolic PDE under noisy Lagrangian sensing. *Arxiv*, 2020. Under review.

Cicic, M. and Johansson, K.H. Stop-and-go wave-dissipation using accumulated controlled moving bottle-necks in multi-class CTM framework. In *Conference on Decision and Control (CDC)*, 2019.