FABLE: Fast Approximate Block Encoding Quantum Circuits

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Introduction

- QSVT: Unification of quantum algorithms.
- Challenge: Ultimately relies on a block encoding circuit for the system matrix.
  - Not immediately clear how to construct these.
  - See: talk by Chao Yang on Thursday for strategies to construct $\mathcal{O}(\text{polylog}(N))$ circuits.

FABLE

FABLE circuits …

- generically contain:
  - N^2 two-qubit CNOTs and N^2 single-qubit R_y gates.
  - $\mathcal{O}(\text{polylog}(N))$ overhead.
- are compressible, in practice often more efficient approximate circuits.
- are *easy* to generate and simulate in QCLAB:

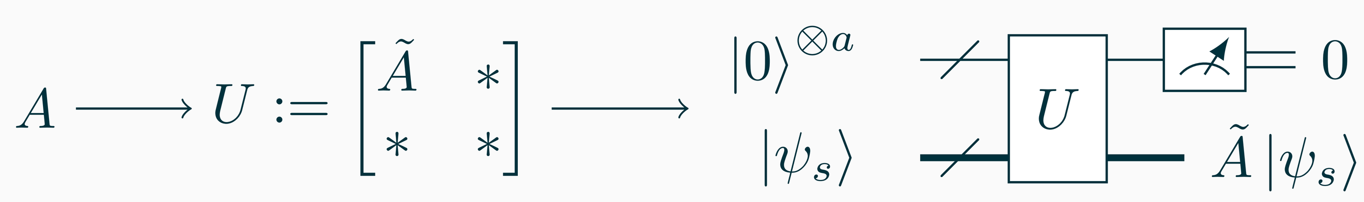
```
[circuit, ~, alpha] = fable( A ) ;
U = circuit.matrix ;
A_BE = 2^n * U(1:2^n, 1:2^n) ;
norm(A - alpha*A_BE)
```

```
ans =

   2.7439e-15
```

Block encodings

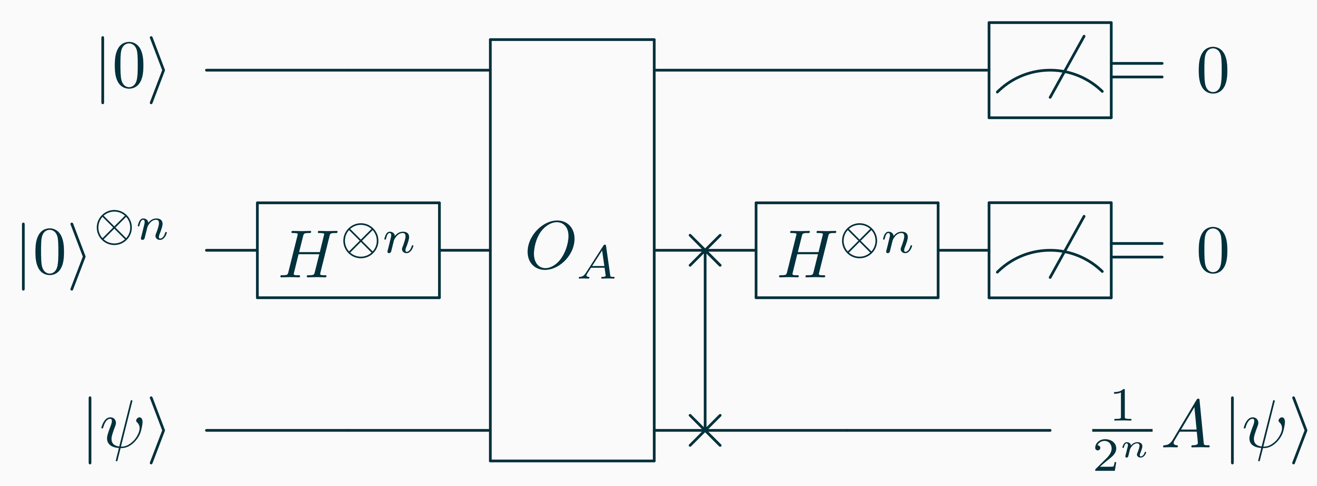
A block encoding approximately embeds a –not necessarily unitary– operator A as the leading principal block in a larger unitary U:



with U an (α, a, ϵ)-block encoding of A where $\|A - \alpha\tilde{A}\|_2 \leq \epsilon$.

Circuit template

- FABLE circuits use the following template:



- O_A is a matrix query oracle:

$$O_A\,|0\rangle\,|i\rangle\,|j\rangle = \left(a_{ij}\,|0\rangle + \sqrt{1 - |a_{ij}|^2}\,|1\rangle\right)|i\rangle\,|j\rangle$$

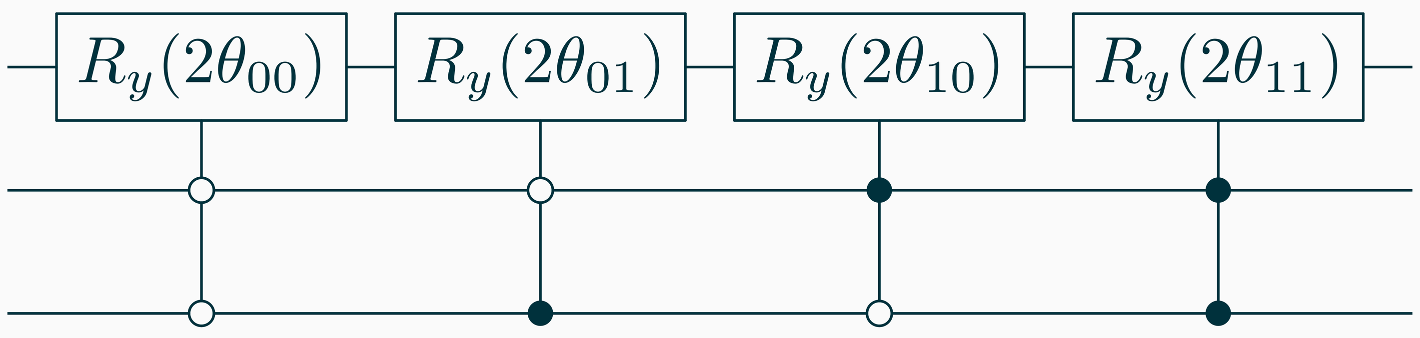
- FABLE solves the problem of synthesizing O_A in simple one- and two-qubit gates for arbitrary matrices A.
- Hadamard and SWAP gates add $\mathcal{O}(\text{polylog}(N))$ overhead.
- The result is an $(1/2^n, n+1, 0)$-block-encoding for an n-qubit matrix A with $|a_{ij}| \leq 1$.

O_A for real-valued data

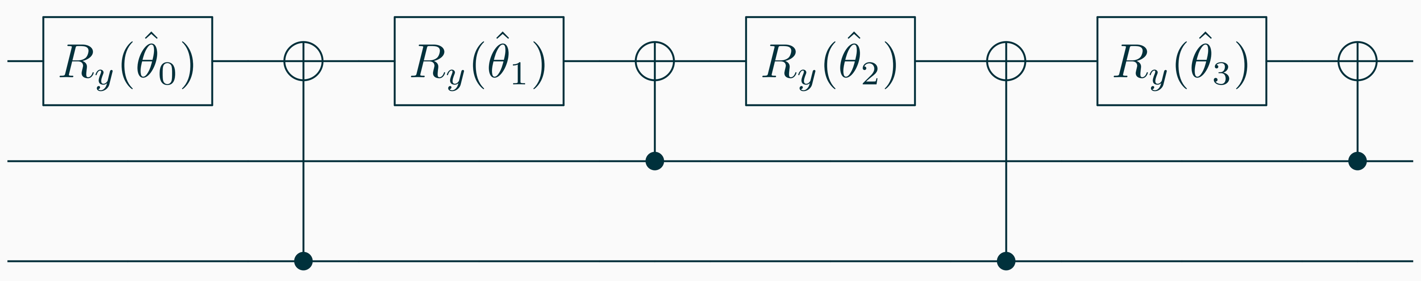
- For an entry a_{ij}, O_A acts as an R_y gate with angle $\theta_{ij} = \arccos(a_{ij})$:

$$R_y(2\theta_{ij})\,|0\rangle = \begin{bmatrix} \cos(\theta_{ij}) & -\sin(\theta_{ij}) \\ \sin(\theta_{ij}) & \cos(\theta_{ij}) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a_{ij} \\ \sqrt{1 - a_{ij}^2} \end{bmatrix}$$

- Naive implementation of O_A requires $\mathcal{O}(N^4)$ elementary gates:



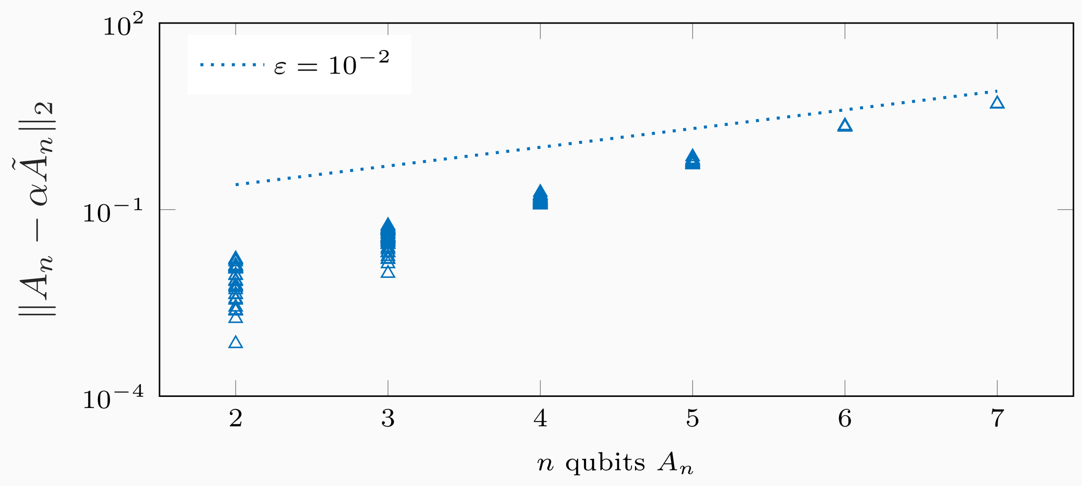
- An improved implementation reduces cost to $\mathcal{O}(N^2)$:



- The angles $\boldsymbol{\theta}$ and $\hat{\boldsymbol{\theta}}$ are related by $\left(\hat{H}^{\otimes 2n} P_G\right)\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}$, which can be solved classically with cost $\mathcal{O}(N^2 \log N^2)$.

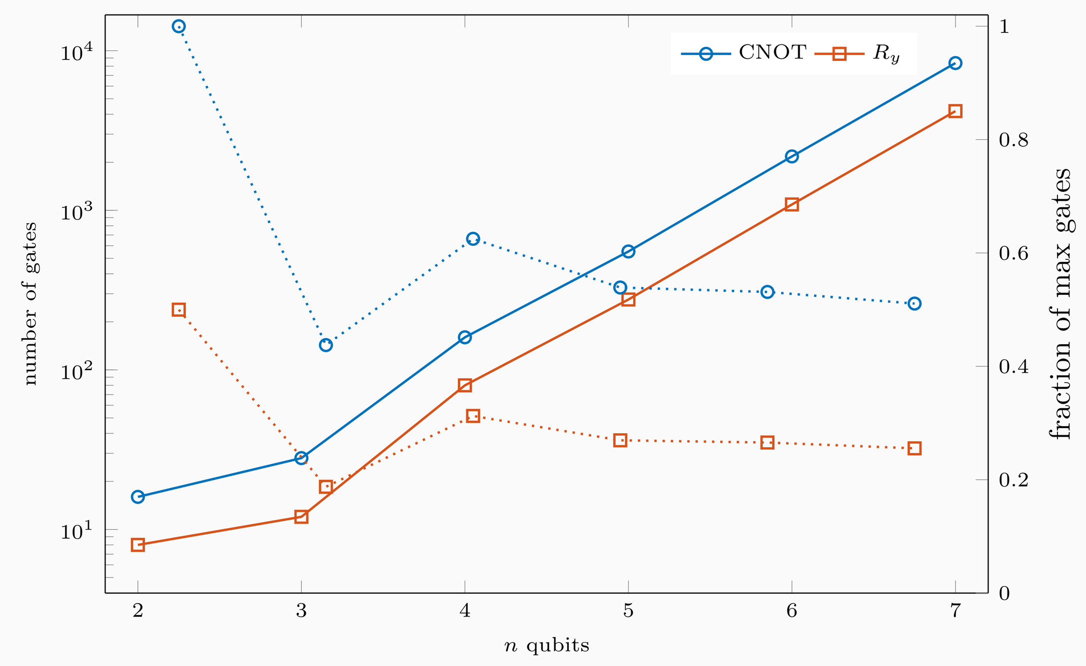
Compression Theorem

For a given cutoff threshold $\varepsilon \geq 0$, setting all angles $|\hat{\theta}_i| \leq \varepsilon$ to zero, results in a $(1/2^n, n+1, 2\pi N\varepsilon)$-block-encoding of an n-qubit matrix A.



Heisenberg XXX Hamiltonian

The Heisenberg XXX model can be encoded w/o compression ($\varepsilon = 0$), with 50% fewer CNOTs and 75% fewer R_y gates:



Conclusion

- FABLE can easily generate and compress block encoding circuits up to ~ 15 qubit operators.
- Encoding of complex-valued data doubles the number of gates.

Reference: Camps D. and Van Beeumen R., *Fast Approximate Quantum Circuit Synthesis for Block Encoding Matrices, In Preparation.*