Noise Induced Delocalization in Strongly Localized Quantum Systems

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Introduction

The quantum diffusion in 1D tight-binding model has an enriched background. L. Hufnagel et al had done a numerical calculation [1] which resulted that the variance of a wave packet in 1D tight-binding can show a superballistic increase ($\sigma^2 = t^\nu$ with $2 < \nu \leqslant 3$) for parametrically large time intervals with the appropriate model. There are other studies for anomalous sub-diffusive, diffusive and super-diffusive dynamics. The delocalization role of the noise in presence of strong localization had been studied by two of the authors in the ref [2]. We have showed that in stongly localized 1D tight-binding systems, Markovian noise can delocalize the system into the desired sub-diffusive, diffusive ,super-diffusive and even superballistic regime.
When the tunneling matrix elements are much smaller than the diagonal elements (deep in the MBL state), one can simplify the Hamiltonian for fermion systems weakly coupled to a bath, by treating the bath as a classical diagonal noise source. Under these conditions, the off-diagonal elements of the density matrix dephase rapidly and the density matrix becomes effectively diagonal after a microscopic time scale. For sufficiently weak coupling, the density matrix has enough time to dephase between successive quantum jumps as the local jump operators (the system operators directly coupled to the bath) do not create resonances deep in the localized phase. One can thus view the density matrix as a classical (joint) probability distribution at all times[3]. Hence, one also can consider bath as classical diagonal noise.
The many-body localization (MBL) transition, strongly depends on the amplitude and the class of disorder pattern of the on-site potential as well as the amplitude of noise and the hoping terms. Gopalakrishnan et.al, [4] speculated that in the limit of Markovian noise there is no intermediate anomalous diffusive regime and the anomalous regime that they found was assumed as a special property of systems coupled to non-Markovian systems. Here we want to point that, this intermediate anomalous regime had already been found in Markovian systems [2] and its not necessary for the on-site energies to be drawn from a Gaussian distribution; It could be quasi-periodic, chaotic binary sequences or even ordered. Here, we elaborate more on our previous findings and how they relate to MBL[5].

The System

Here, we look into similar system of non-interacting fermions in one dimension, subject to a static disorder on-site energies and pure dephasing white-noise[6],

$$H = -J \sum_{<i,j>} c_i^\dagger c_j + \epsilon_i \sum_i c_i^\dagger c_i \quad (1)$$

where $J = 1$ represents the tunneling matrix elements, where $i = j \pm 1$ and $c_i^\dagger$ (c_i) is the particle creation (annihilation) operator on lattice site i. The on-site energies ϵ_i are drawn from a uniform distribution over $[1, 3)$. Through utilizing the density matrix $\rho(t)$, one can determine the time evolution of an open system by the Lindblad equation:

$$L(\rho) = \frac{1}{2} \sum_j ([\Gamma_j \rho, \Gamma_j^\dagger] + [\Gamma_j, \rho \Gamma_j^\dagger]) - \frac{i}{\hbar}[H, \rho] \quad (2)$$

where $\Gamma_j = \gamma jj$ is our noise operator with amplitude $\gamma \in \{0.01, 0.02, 0.04, 0.08, 1.0, 1.5\}$. Since the diagonal terms (H_{ii}) and the noise terms (Γ_j) are not the dominating energy scales, the coherences become the critical parameters of our problem and there is no way to eliminate them adiabatically. Therefore, it is not possible to map the quantum evolution to classical rate equations and one could not generalize the results from the systems deep in the localized state to the current system.
We utilize the variance of the wave packet to account for it's spreading

$$\sigma^2 \equiv \sum_n n^2 |\psi_n|^2 \quad (3)$$

where n is the lattice site index and ψ_n is the normalized time-evolving wave packet. We use numerical calculation[7][8][9][10][11] for solving the corresponding Lindblad equation for a particle localized in the center of a system of 400 sites.

The Spreading of the Wavepacket

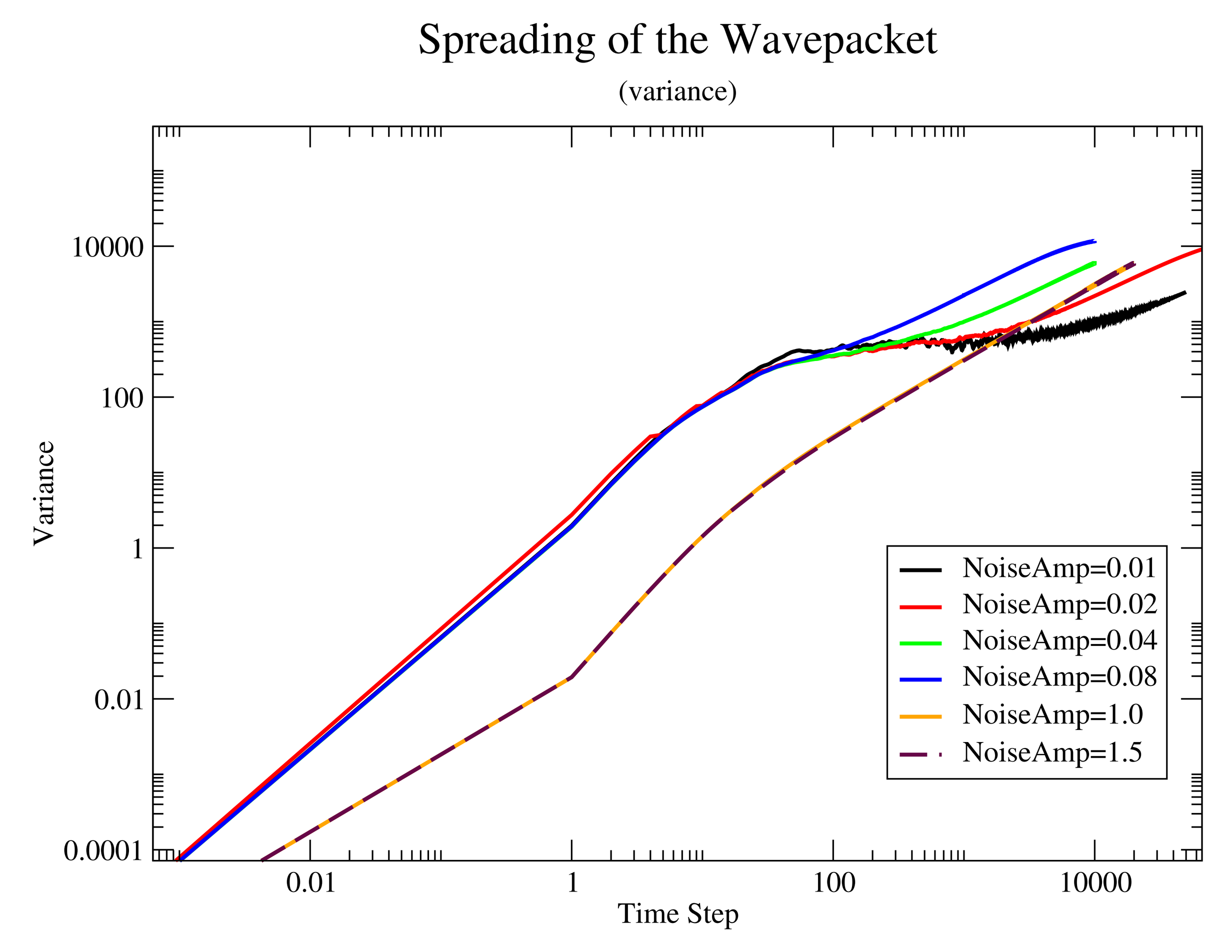

Figure 1. Time dependency of the spread $\sigma^2 \sim t^\beta$ of an initially localized wave-packet for systems of size $L = 400$ for different noise amplitudes. The results draw from 20 different disorder realizations.

As it can be seen in Fig. 1 and Fig. 2, we use different number of time steps for different noise values. For $\gamma = 1.0$ and 1.5, as the noise amplitude is large, the diffusion power reaches its stable value ($\beta = 1$) after around 100 steps, and at around the step 10^4, β starts to decrease until it gets zero, due to the finite size effect. One can see immediately that for $\gamma = 1.0$ and $\gamma = 1.5$, the diffusion power's stable value correspond to the diffusive regime and there is no anomalous sub-diffusion in between the ballistic regime and the diffusive one, which is in agreement with the theoretical finding of Ref[3]. By decreasing the noise amplitude, the diffusion power β reaches its stable value at later steps and the coherence elements of the density matrix become more important. For the noise amplitude $\gamma = 0.01$, due to computational limitation, we could not reach the step at which the diffusion power become stable (if exists). In our Markovian system for some noise amplitudes, there is no crossover to diffusion; Fig. 2.

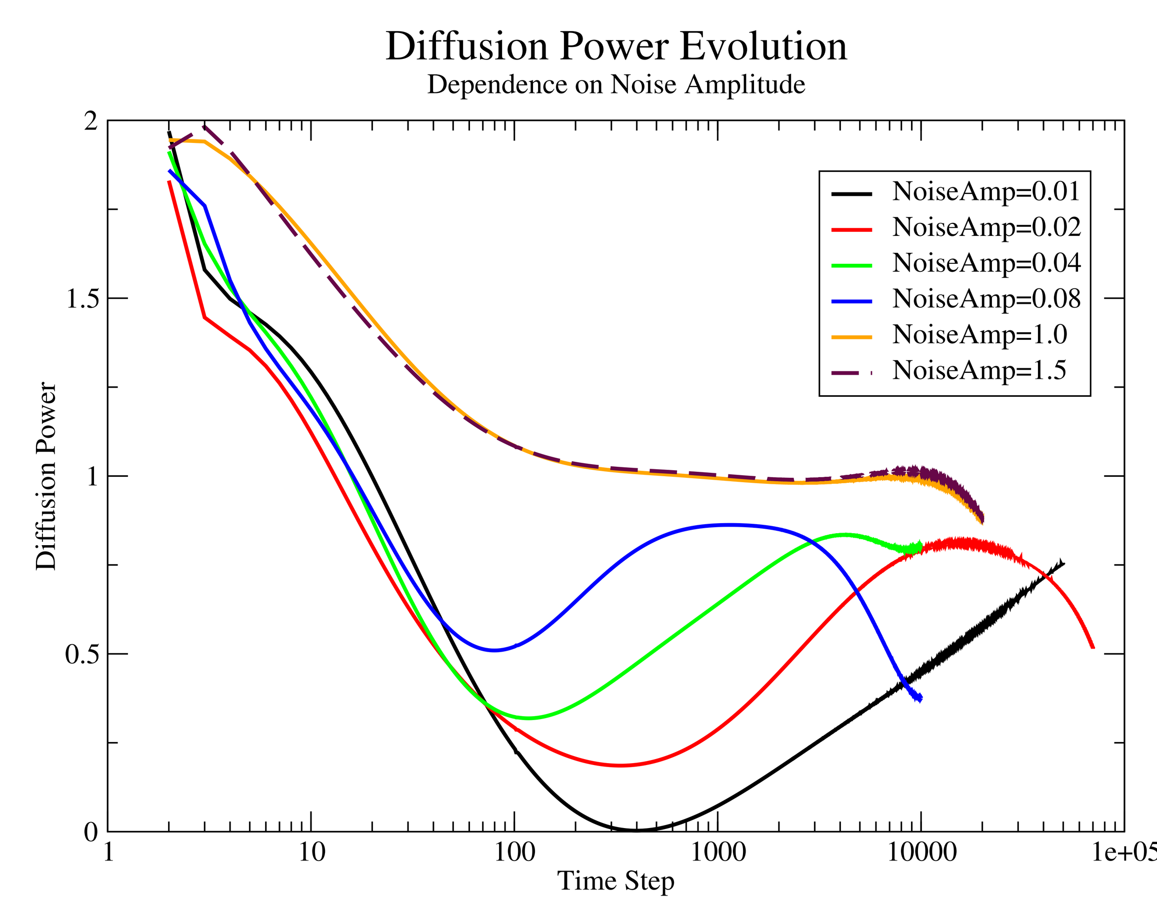

Figure 2. Time dependency of the power β of wave-packet variance $\sigma^2 \sim t^\beta$ of an initially localized wave packet for the same systems as Fig. 1.

As we can see in the figure in a Markovian system, an anomalous sub-diffusive regime could exist after the ballistic regime, and when the noise amplitude is low enough ($\gamma \in \{0.02, 0.04, 0.08\}$), sometimes there is no crossover to diffusion.

Future Direction and the Final Remarks

This problem not only can contribute to the physical difficulties in implementation of quantum computer, especially using plasmons. We need more insight on MBL phases there. This type of systems can be used to design quantum system co responding to nonlinear ODEs, which would be easier than the real quantum computer. Moreover there is already possible suitable practical implementable chances in Ion traps, etc. Other interesting direction is looking at the conduction in such system and the coherent dynamics in these systems.

References

[1] L. Hufnagel, R. Ketzmerick, T. Kottos, and T. Geisel. Superballistic spreading of wave packets. *Phys. Rev. E*, 64:012301, Jun 2001.

[2] Ehsan Gholami and Zahra Mohammaddoust Lashkami. Noise, delocalization, and quantum diffusion in one-dimensional tight-binding models. *Phys. Rev. E*, 95:022216, Feb 2017.

[3] Mark H Fischer, Mykola Maksymenko, and Ehud Altman. Dynamics of a many-body-localized system coupled to a bath. *Phys. Rev. Lett.*, 116:160401, Apr 2016.

[4] Sarang Gopalakrishnan, K. Ranjibul Islam, and Michael Knap. Noise-induced subdiffusion in strongly localized quantum systems. *Phys. Rev. Lett.*, 119:046601, Jul 2017.

[5] Zahra Mohammaddoust Lashkami, Younes Younesizadeh, and Ehsan Gholami. Comment on noise-induced subdiffusion in strongly localized quantum systems, 2021.

[6] H. Haken and G. Strobl. An exactly solvable model for coherent and incoherent exciton motion. *Zeitschrift für Physik A Hadrons and nuclei*, 262(2):135–148, Apr 1973.

[7] Pauli Virtanen, Ralf Gommers, Travis E. Oliphant, Matt Haberland, Tyler Reddy, David Cournapeau, Evgeni Burovski, Pearu Peterson, Warren Weckesser, Jonathan Bright, Stéfan J. van der Walt, Matthew Brett, Joshua Wilson, K. Jarrod Millman, Nikolay Mayorov, Andrew R. J. Nelson, Eric Jones, Robert Kern, Eric Larson, C J Carey, İlhan Polat, Yu Feng, Eric W. Moore, Jake VanderPlas, Denis Laxalde, Josef Perktold, Robert Cimrman, Ian Henriksen, E. A. Quintero, Charles R. Harris, Anne M. Archibald, Antônio H. Ribeiro, Fabian Pedregosa, Paul van Mulbregt, and SciPy 1.0 Contributors. SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python. *Nature Methods*, 17:261–272, 2020.

[8] Charles R. Harris, K. Jarrod Millman, Stéfan J van der Walt, Ralf Gommers, Pauli Virtanen, David Cournapeau, Eric Wieser, Julian Taylor, Sebastian Berg, Nathaniel J. Smith, Robert Kern, Matti Picus, Stephan Hoyer, Marten H. van Kerkwijk, Matthew Brett, Allan Haldane, Jaime Fernández del Río, Mark Wiebe, Pearu Peterson, Pierre Gérard-Marchant, Kevin Sheppard, Tyler Reddy, Warren Weckesser, Hameer Abbasi, Christoph Gohlke, and Travis E. Oliphant. Array programming with NumPy. *Nature*, 585:357–362, 2020.

[9] J. D. Hunter. Matplotlib: A 2d graphics environment. *Computing in Science Engineering*, 9(3):90–95, 2007.

[10] S. Behnel, R. Bradshaw, C. Citro, L. Dalcin, D. S. Seljebotn, and K. Smith. Cython: The best of both worlds. *Computing in Science Engineering*, 13(2):31–39, 2011.

[11] P. Brown, G. Byrne, and A. Hindmarsh. Vode: a variable-coefficient ode solver. *Siam Journal on Scientific and Statistical Computing*, 10:1038–1051, 1989.

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