Quantum Numerical Linear Algebra @ IPAM Open Problem Session
January 27, 2022, 12:45-1:15pm PST

Description: During the Quantum Numerical Linear Algebra workshop, participants contributed to this document as open problems in quantum algorithms were discussed during a 30 minute session.

Fault-tolerant quantum algorithms for linear algebra

Ground state preparation, initial guess.
Efficient use of initial density matrix / purified state for preparing the Gibbs state (contrast between classical and quantum Hamiltonians)
Solve graph Laplacian on QC?

Fault-tolerant quantum algorithms for differential equations

Imaginary time evolution, multiplicative error. (https://arxiv.org/abs/1612.05602 Lemma 1). This is useless for unitary time-evolution. But is there a benefit to this for linear systems problem or differential equations?
Hamiltonian simulation: unbounded Hamiltonians?
Simulate open systems.
Commutator bounds for high order Trotter, they do not hold for time dependent simulation
Multiple different ways for representing quantum dynamics, such as path integral formulation, Bohemian mechanics etc. Simulating QFT with path integrals on QC.
Combine various HS methods to take advantage of structural features.
Hybridize HS with QEC.
Simulate a large number of Hamiltonian ODEs?

Transport equation (non-dissipative equations)
Algorithms for other types of dispersive equations
Classical information, input and output.

A demonstration of quantum advantage for non-linear differential equations without resorting to the argument that it contains linear systems which is BQP-complete.

Quantum version of “nonlinear” Monte Carlo methods (Branching process. Refs?)
Exponential speedup over Monte Carlo methods? (“sign problem”)
Stronger nonlinearity versus smaller initial condition, not the chaotic regime.

Near term quantum algorithms for linear algebra

Barriers of variational linear system solvers. Verify some small part of the solution (expectation value), is it possible to get around the lower bound?

Using linear algebra as “application based benchmarks”

Parallel algorithms

Trade off between circuit depth and width (e.g., parareal algorithm for classical differential equations)

Parallel speedup for Hamiltonian dynamics (e.g., https://arxiv.org/abs/2105.11889)? Depth-width tradeoff, input model (QRAM). Costs are in “magic states”, embarrassingly parallel?

Parallelization of the output of phase estimation algorithms.

No go theorems? Circuit lower bound (Microsoft group “Haah-Hastings-Kothari-Low”), depth bound, no-fast-forwarding.

Distributed / multi-agent machine learning?

1. Solving linear algebra problems associated with unbounded operators.
2. “Poster-child” problem for establishing exponential quantum advantage for linear algebra problems.
3. Understand when non-linear algebra / polynomial equations / differential equations can be efficiently solved on quantum computers.
4. Find ways of sidestepping the “input problem”

Analog quantum simulation
https://pubs.rsc.org/en/content/articlepdf/2021/sc/d1sc02142g

Paper links posted in the chat:

https://arxiv.org/abs/2105.11889

https://arxiv.org/abs/1909.10503 / https://arxiv.org/abs/1909.10303

https://journals.aps.org/pra/abstract/10.1103/PhysRevA.102.022607

https://arxiv.org/pdf/1612.05602.pdf